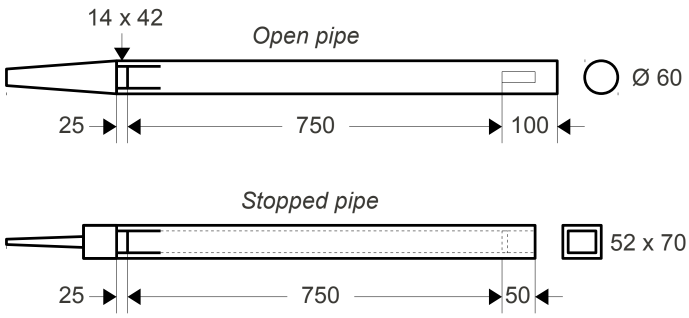
Introduction

During the construction of an electronic organ a theory for the acoustical output from the flue pipe was required; however, the available literature did not give useful models so an algorithm was build using acoustical theory and published responses measured on real pipes. The result is surprisingly simple – using two parameters only: the physical pipe length *L* defines the fundamental frequency of oscillation and the ratio of length to diameter *L/D* defines the timbre of the pipe. This is not the full story about the spectrum of an organ pipe but the result is anyway promising.



Appendix-FluteDimensions

The top drawing represents an open pipe made of thin-walled metal with circular cross section. The pipe length is measured from the sharp edge at the labium to the edge of the large tuning hole or to the open end. The pipe is open (half wavelength) and sounds at 200 Hz and the ratio L/D = 13 identifies a pipe belonging to the flute-family characterised by few harmonics. The lower drawing is a wooden stopped pipe with tuning through the movable stopper. The material is 10 mm thick with inner dimensions shown. The pipe is stopped (quarter wavelength) sounding at 110 Hz. The ratio L/D = 11 corresponds to L/D = 22 for an open pipe so the pipe belongs roughly to the same family as the open pipe although with higher-pitched harmonics; but the even harmonics are missing due to the resonance at quarter wavelength.

Reported spectra for pipes are used as reference for the model but lack of data necessitated the use of assumptions. The mechanical dimensions were not reported for the pipes with published responses and pipes with mechanical measures were not equipped with recorded spectra. An additional complication was that lots of erroneous material is available through the internet so quite some detective work was required in order to succeed.

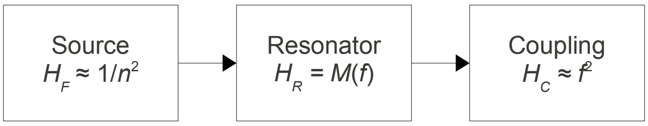
A note on modelling organ pipes

The real pipe is not characterised solely by the pipe spectrum and regardless of how precisely the electronic organ models the spectrum the perception will not be that of an organ pipe. A real pipe is characterised by the growing speed of the individual harmonics following key down; such as third harmonic starting before the fundamental. The pitch is dependent upon air pressure so the pitch decreases when several keys are pressed for pipes on the same wind chest. Noise from the organ mechanics is part of the starting sound but is hard to model electronically and turbulence noise is shaped by pipe resonance to become part of the resulting sound. Finally, the physical size of the organ spreads sound into three dimensions before being added at the ear canal and this cannot be modelled electronically before the sound is output acoustically since perception is dependent upon the position of the listener. These factors are not addressed by the present study yet they are of major importance for the perception of the resultant sound – so you will not succeed modelling the grand organ by electronic means; however, the result can be well-sounding and pleasing.

Contents

The building blocks of an organ pipe

An organ pipe is energised from an external air supply with an oscillating air stream; an *air reed* called *the flue* generating an *edge tone* and oscillation builds up within the resonator until limited by the losses balancing the input power. Part of the oscillating air pressure within the pipe is output as audible sound through radiation from the open ends.



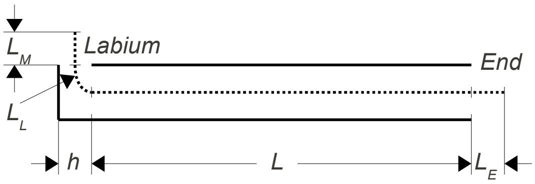
Appendix-FluteBlocks

The sound is generated by an oscillating air stream, which is slightly distorted thus containing harmonics. The flue drives the resonator at the available oscillation modes and the pressure within the pipe is build up. A fraction dissipates into the surroundings as audible sound.

The following description assumes an open-ended pipe with constant cross-sectional area, which represents the typical organ pipe and the stopped pipes are only rudimentary covered. The model can be used for stopped pipes through moderate changes within the code.

Resonator

Sound is propagating within the pipe at the speed of sound and is reflected at pipe ends where the acoustical impedance undergo a sudden change. The sign of the travelling wave front is inverted upon reflection from an open end and the reflected wave front must be in-phase with the flue to sustain the signal so two times reflection and full wavelength delay is required for the open pipe, which is satisfied with a physical pipe length of one half wavelength $L = \lambda/2$; hence, the natural frequency of the pipe approximates $f_1 = c/\lambda = c/2L$. For the stopped pipe the reflected signal is not inverted at the closed end so the reflected signal needs one-half wavelength of delay and this is reached with a physical length of one-quarter wavelength, $L = \lambda/4$ and thus $f_1 = c/4L$.



Appendix-FluteLength

The path length from mouth to open end includes the end correction L_M at the mouth and the end correction L_E at the open end plus the extra length L_L from centre of the pipe to the side.

The labium is located at the side of the pipe so the average signal path from the centre of the pipe to the side must be included. With cut-up h and pipe radius a and assuming that the bending can be approximated by an straight line the additional path length is determined using Pythagoras. It is further assumed that the width of the labium is $w \approx a$ and that the cut up is $h \approx a/2$.

$$L_L \approx \sqrt{a^2+h^2} = \sqrt{a^2+\left(\frac{a}{2}\right)^2} = a\frac{\sqrt{5}}{2} \approx 1.12\,a$$

The travelling wave front is not reflected right at the open end but from a point somewhat outside the physical end since the air spreads gradually upon exit from the pipe so the pressure does not immediately drop to a lower value corresponding to an additional mass of air, which is commonly described as the *end correction* L_E with the expression derived later in this document.

$$L_E^{Circular} \approx 0.60\,a \quad \text{or} \quad L_E^{Rectangular} \approx 0.60\sqrt{\frac{l_1 l_2}{\pi}} \quad \text{where} \quad \begin{array}{l} a \text{ is radius of the cylinder} \\ l_1 \text{ and } l_2 \text{ are side lengths} \end{array}$$

The mouth is approximately rectangular and using $l_1 = w \approx a$ and $l_2 = h \approx a/2$ the end correction for the mouth becomes:

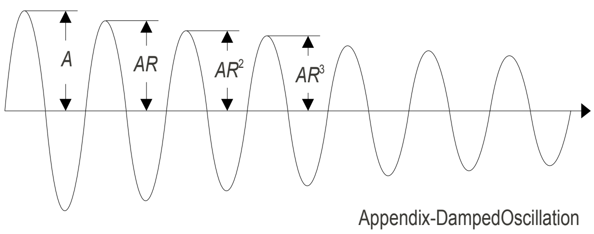
$$L_M \approx 0.6\sqrt{\frac{hw}{\pi}} \approx 0.6\sqrt{\frac{a^2}{2\pi}} \approx 0.24\,a$$

An open pipe uses side correction L_L plus end corrections L_M and L_E corresponding to the resultant correction of 1.96a, which will be rounded into $D = 2a$. A stopped pipe uses side correction L_L plus the end correction L_M of the mouth into 1.36a, which will be rounded to the correction 0.7 D; hence the following equations for the fundamental frequency:

$$f_1^{Open} = \frac{c}{2\,L + D} \quad \text{and} \quad f_1^{Stopped} = \frac{c}{4\,L + 0.7\,D}$$

Resonance at higher frequencies (*modes*) is possible if the reflections arrive in phase to sustain the oscillation and this is nearly at integral frequencies of f_1, so $f_2 \approx 2f_1$ and $f_3 \approx 3f_1$ and so forth for an open pipe and only odd numbers for stopped pipes. The end correction is frequency dependent but this has little importance on the timbre since higher modes have lower quality factor and are thus less selective; hence, this effect will be ignored.

If the input air stream is interrupted the oscillation dies away as a damped sine oscillation since the loss mechanisms absorbs energy from the mechanical oscillation within the pipe. The decay time is a measure of the mechanical losses and is the target for the discussion to follow. The system is almost perfectly linear so the theory of superposition is valid; i.e. the individual oscillation modes can be studied one at a time.



Once started the oscillation dies away due to losses, which is here represented by the reflection coefficient R. The amplitude A is in the illustration attenuated by factor R for each full wavelength.

A resonant circuit of this kind is described by the Laplace transform through the transform pair shown below where $H(s)$ is the frequency domain representation of the identical twin $h(t)$ within the time domain [5-332]. Note that constants A and B both have dimension of s^{-1} so the equation for $H(s)$ is not dimensionally neutral although the equation for $h(t)$ within the time-domain is.

$$H(s) = \frac{B}{(s+A)^2 + B^2} \quad \Leftrightarrow \quad h(t) = \exp(-At)\sin(Bt)$$

Substituting $B = \omega_1$ for the oscillation frequency and $A = \delta_1\omega_1$ for the damping coefficient gives:

$$H(s) = \frac{\omega_1}{(s+\delta_1\,\omega_1)^2 + \omega_1^2} = \frac{\omega_1}{s^2 + 2\,\delta_1\,\omega_1\,s + (1+\delta_1^2)\,\omega_1^2} \quad \Leftrightarrow \quad h(t) = \exp(-\delta\,\omega_1 t)\sin(\omega_1 t)$$

The Laplace transform variable is $s = i\omega = i2\pi f$ along the imaginary axis so $s^2 = -\omega^2 = -(2\pi f)^2$ and the real terms cancel at the resonance frequency with the imaginary part defining the amplitude. The resonance frequency is almost identical to f_1 for low damping ($\delta_1 < 0.1$). The equation applies to any oscillation mode with the parameters substituted by δ_n and ω_n where n is mode number, so the combined action is described through summation of the modes [2-71]. Multiplication with f_1 results in a dimensionally neutral magnitude function without changing the relative behaviour.

$$M_R(f) = \sum_{n=1}^{N} \left| \frac{f_n f_1}{(1+\delta_n^2) f_n^2 - f^2 + i\,2\,\delta_n f_n f} \right| \quad \text{where} \quad f_n \approx n f_1$$

This description assumes that there is only one direction of sound propagation within the pipe, but signals oscillating along the circular surface may exist when the circumference $2\pi a$ is larger than wavelength $\lambda = c/f$, which is written as $ka > 1$ and radiation becomes directive above this limit but the effect is less than 5 dB for $ka < 1.5$ so the sound is output with nearly equal sound pressure in all directions within the useful frequency range and a suitable limit becomes $ka \approx \pi/2$.

$$ka = \frac{2\pi a}{\lambda} = \frac{2\pi f a}{c} = \frac{\omega a}{c} \quad \text{hence} \quad ka = \frac{\pi}{2} \quad \Rightarrow \quad f_{MAX} = \frac{c}{2D}$$

For pipe radius 25 mm (D = 50 mm) the upper frequency limit for the model becomes 3.4 kHz and using the (approximate) expression of the fundamental frequency the number of modes within the open pipe approximates 50 for an 8' pipe [3-240]:

$$N = \frac{f_{MAX}}{f_1} \approx \frac{c/2D}{c/2L} = \frac{L}{D}$$

Mode amplitude is determined through the damping coefficient δ_n, which is given by the total loss within the system. According to the Laplace transform pair the coefficient $\delta_1\omega_1$ defines the rate of decay through $\exp(-\delta_1\omega_1)$ for the fundamental oscillation where $t = T_1 = 1/f_1 = 2\pi/\omega_1$ and the signal has propagated from the mouth to the end and back again to the mouth, i.e. two times pipe length.

With p_0 for the initial sound pressure and p_1 for the pressure after one period ($f_1 = 1/T$) the equation below shows the attenuation due to individual loss mechanisms. Using the rule for multiplication of exponentials the equation can be simplified into the addition of damping coefficients.

$$p_1 = p_0 \prod_i \exp(-\delta_i \omega_1 T) = p_0 \prod_i \exp(-\delta_i 2\pi) \quad \Rightarrow \quad A = \frac{p_1}{p_0} = \exp(-2\pi \sum_i \delta_i)$$

Reflection

Almost all signal is reflected at the end of the pipe with but a fraction of the signal being transmitted into the environment. This is described through the reflection coefficient R, which is defined from the radiation impedance Z_A at the end of the pipe end and is shown graphically below [1-122].

The acoustical impedance inside the pipe is $\rho_0 c/S$ where $\rho_0 c$ = 412 kg m^{-2} s^{-1} is the characteristic impedance of air and $S = \pi a^2$ is the cross-sectional area of the pipe. The expression assumes the propagation of plane waves so the wavelength must be long compared to radius.

$$R = \frac{Z_A - \rho_0 c/S}{Z_A + \rho_0 c/S} = \frac{z-1}{z+1} \quad \text{where} \quad z = \frac{S Z_A}{\rho_0 c} \quad \text{and} \quad ka < 2$$

The acoustical impedance is required for calculation but the mechanical impedance is reported within the reference so a relation between the two is derived (commented below).

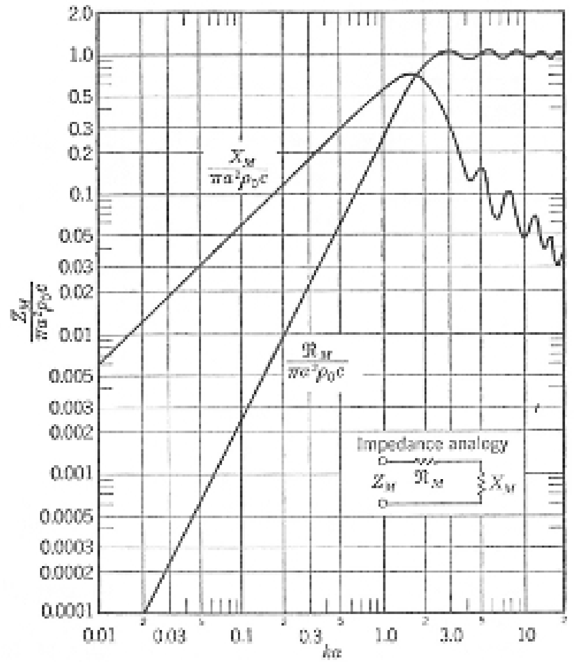
$$F = Z_M v, \quad F = Sp, \quad p = Z_A q, \quad q = Sv \quad \Rightarrow \quad Z_M = \frac{F}{v} = \frac{Sp}{v} = \frac{S^2 p}{q} = S^2 Z_A$$

> The mechanical impedance Z_M is given by force F and velocity v, with force being pressure p working on area S, i.e. the air pressure difference between the two sides of a diaphragm. The acoustical impedance Z_A is defined by pressure p and volume velocity q where the latter is the volume of air being moved at speed v. Moving the diaphragm with area S through the distance x moves volume $V = Sx$ and if moved at velocity $v = dx/dt$ the volume velocity is $q = dV/dt = Sv$.

Using $SZ_A = Z_M/S$ with pipe area $S = \pi a^2$ the normalised impedance $z = x + iy$ can be written as follows where the right-hand terms are defined by the figure above. Two empirical approximations for x and y are shown within the figure and are used within the software.

$$z = \frac{S Z_A}{\rho_0 c} = \frac{Z_M}{S \rho_0 c} = \frac{Z_M}{\pi a^2 \rho_0 c} = x + iy = \frac{R_M}{\pi a^2 \rho_0 c} + \frac{i X_M}{\pi a^2 \rho_0 c}$$

The acoustical output is due to the real part, which is proportional to frequency squared so the pipe is an inefficient radiator of the lower harmonics; the transfer function corresponds to a high-pass filter of second order with cut-off at approximately $ka = 2$ so the upper limit is $f \approx c/\pi a$.



Approximations:

$$R_M: \ x = \frac{\left(\frac{ka}{2}\right)^2}{\sqrt{1+\left(\frac{ka}{2}\right)^4}}$$

$$X_M: \ y = \frac{\frac{ka}{1.7}}{\sqrt{1+\left(\frac{ka}{1.7}\right)^6}}$$

Within ± 1 dB, $ka < 4$

The radiation impedance acting upon a circular piston at the end of a long cylindrical tube. The vertical axe is normalised using $\pi a^2 \rho_0 c$, where a is radius of the piston, ρ_0 = 1.2 kg/m^3 is the mass density of air and c = 343 m/s is the speed of sound (from Beranek “Acoustics”).

The imaginary part is directly proportional to frequency and can be written as $X_M = \omega M_M$ where the mechanical mass M_M is air outside the pipe oscillating along with the air within the pipe and this appear as an extra load at the end of the pipe. The impedance is $X_M = 0.6\, \pi a^2 \rho_0 c$ for $ka = 1$ as per figure and since $ka = \omega a/c$ we have $\omega = c/a$ at this point and the mechanical mass becomes:

$$X_M = \omega M_M = 0.6 \pi a^2 \rho_0 c \quad \text{at} \quad ka = 1 \quad \Rightarrow \quad M_M = 0.6 \pi a^3 \rho_0$$

Considering the additional mass as a a cylinder of air with radius a and length L_E the mass is given by the mass density of air multiplied by the volume of air $M_M = \rho_0 \pi a^2 L_E$ and equating the equations for the mechanical mass gives an expression for the end correction of a long and cylindrical pipe, which was referenced earlier and is one of the quoted expressions[1] within the literature.

1 An often quoted expression is $L_E = 0.85\ a$ [2-167], which applies to an opening flush with an infinite baffle and is often seen within articles of loudspeaker design.

$$M_M = \pi a^2 \rho_0 L_E \quad \Rightarrow \quad L_E = \frac{M_M}{\pi a^2 \rho_0} = \frac{0.6 \pi a^3 \rho_0}{\pi a^2 \rho_0} \quad \Rightarrow \quad L_E = 0.6\, a$$

At the open end the reflection coefficient R_1 is calculated using the cross-sectional area of the pipe. The reflection coefficient R_2 at the mouth uses a smaller area given by the width and height of the labium, *w* and *h* respectively. The area will be expressed as radius *b* of a circular opening with the same area, and since the width is approximately equal to pipe radius ($w \approx a$) while the height is smaller ($h \approx a/2$) the typical pipe design gives radius *b* to some 40 % of *a*.

$$S = wh = \pi b^2 \quad \Rightarrow \quad b = \sqrt{\frac{wh}{\pi}} \approx \frac{a}{2.5} \quad \Rightarrow \quad kb \approx \frac{ka}{2.5}$$

Attenuation is R_1 when the wave front is reflected from the open end and R_2 when reflected from the mouth so the signal is reduced by R_1R_2 for one period at the fundamental frequency. Using the definition of the damping coefficient the following equation is derived. Note that the logarithm to an argument less than unity is negative so the quantity δ_n is indeed positive.

$$\exp(-\delta_R 2\pi) = |R_1||R_2| \quad \Rightarrow \quad \delta_R = -\frac{\ln(|R_1||R_2|)}{2\pi}$$

Using the frequency f = 220 Hz and pipe radius a = 25 mm we find ka = 0.10 where R_M = 0.0025 and X_M = 0.06 at the open end, and ka = 0.04 at the labium where R_M = 0.0004 and X_M = 0.025. Ignoring the imaginary parts: R_1 = –0.9975/1.0025 = –0.995 and R_2 = –0.9994/1.0004 = –0.999. The damping coefficient is $\delta_R = \ln(0.995 \cdot 0.999)/2\pi = 955 \cdot 10^{-6}$ and increases with frequency.

Friction

The model uses the Rayleigh model [2-180], which applies to narrow tubes and this fits well to the project at hand. It is commonly included into the equation for wave propagation shown below with $i\omega_1 t$ representing the phase of the signal as time goes by and ik_1x the phase due to the distance along the pipe. The attenuation is introduced through adding an imaginary term to the propagation constant k thus introducing a real value into the equation (since i^2 is real). The square root is simplified using Taylor and the air viscosity is $\eta = 1.8 \cdot 10^{-5}$ kg m^{-1} s^{-1}.

$$p = p_0 \exp(i\omega_1 t - ik_1 x) \quad \text{where} \quad k_1 = \frac{\omega_1}{c}\sqrt{1 - \frac{i 8\eta}{\rho_0 \omega a^2}} \approx \frac{\omega_1}{c}\left(1 - \frac{i 4\eta}{\rho_0 \omega a^2}\right)$$

The propagation distance for one period is $x \approx 2L$ (ignoring the corrections) and the fundamental frequency is $f_1 = \omega_1/2\pi \approx c/2L$ (see page 3). The real term represents the damping due to friction and the imaginary part represents the oscillation frequency with a delay of one period.

$$p = p_0 \exp\left(i\omega_1 t - i\frac{\omega_1}{c}\left(1 - \frac{i4\eta}{\rho_0 \omega a^2}\right)2L\right) = p_0 \exp\left(i\omega_1\left[t - \frac{2L}{c}\right] - \frac{4\eta\omega_1}{\rho_0 \omega c a^2} 2L\right)$$

The damping coefficient becomes as follows using $2f_1L \approx c$.

$$-2\pi\delta_V = -\frac{8\eta\omega_1 L}{\rho_0 \omega c a^2} \quad \Rightarrow \quad \delta_V = \frac{4\eta f_1 L}{\pi \rho_0 f c a^2} \approx \frac{2\eta}{\rho_0 \pi a^2 f}$$

Using f = 100 Hz and a = 25 mm the value of the damping coefficient is $\delta_V = 153 \cdot 10^{-6}$ and since it decreases with frequency this parameter is significant primarily to the lower harmonics.

Air absorption

The attenuation is proportional to the travelled distance with the constant *m*. The travelled distance for one full period is two times pipe length ($x = 2L$).

$$\frac{p}{p_0} = \exp(\frac{-mx}{2}) = \exp(-mL) = \exp(-2\pi\delta_A) \quad \Rightarrow \quad \delta_A = \frac{mL}{2\pi}$$

Table values of the constant *m* [2-163] is approximated through the equation shown below. The air absorption is not particularly important and need not to be modelled carefully since it is an order of magnitude below the former two contributions.

$$m \approx [0.64 + 0.31\,(\frac{f}{\text{kHz}})^2]\cdot 10^{-3}\,\text{m}^{-1}$$

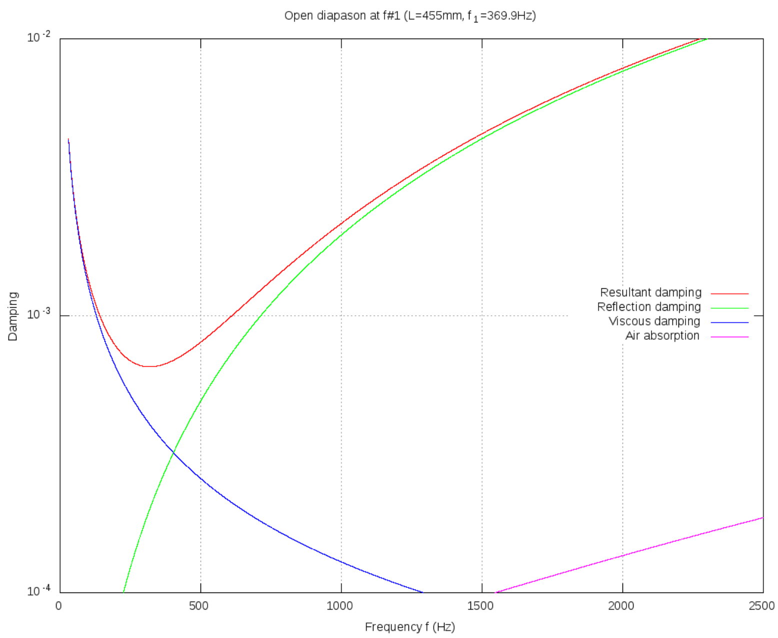
With L = 1.56 m for f_1 = 110 Hz we find $m = 0.64\cdot10^{-3}\ \text{m}^{-1}$ and thus $\delta_A = 160\cdot10^{-6}$ and the value increases with frequency. Air absorption is of lesser significance although it may increase to approximately 30 % of the resultant damping for some of the pipes.

The resultant spectrum

The resultant damping is calculated as the sum of the contributions with an example shown below as the red line consisting primarily of reflections at pipe ends from the second harmonic and up (green) while the viscous damping dominates at low frequencies (blue).

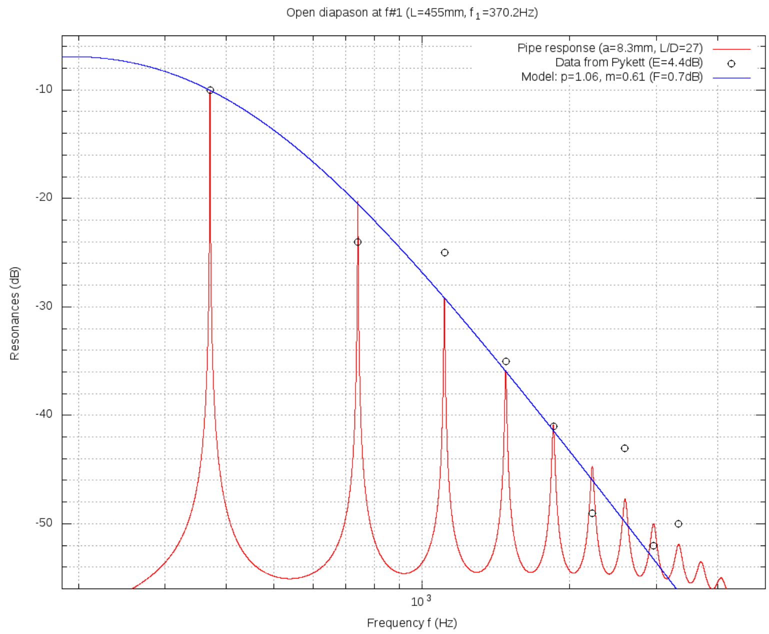
$$\delta_n = \delta_R + \delta_V + \delta_A$$

The bandwidth of the resonance is $B_1 = f_1/Q_1 = 2f_1\delta_1$. The damping is $\delta_1 \approx 0.00066$ at $f_1 \approx 370$ Hz so the quality factor is $Q_1 \approx 760$ and the bandwidth is $B_1 \approx 0.5$ Hz between the –3 dB marks.



Damping coefficients for a diapason pipe with length 455 mm and radius 8.6 mm.

An example of the simulated response for the same pipe is shown below as the red line together with reported data from measurement of the pipe shown as circles. The average result is good so the simulated resonance spectrum is expected to approximate the pipe response, which is a very important result and valuable input to the sections to follow. The differences may be due to the pipe construction details (i.e. not simulated) or the test set up may have influenced the response due to acoustical reflections within the propagation path. The absolute value of the resonance (–10 dB) is immaterial and only the relative response should be considered.



Simulated response of an open diapason pipe shown as the red line. Reported measurements are shown as black circles and the blue line is an approximation to be described later.

Coupling

The coupling from resonator to environment is proportional to frequency squared due to the real part of the radiation impedance and is almost constant at higher frequencies (ignoring the ripples) thus representing a second-order high-pass filter with cut-off at $ka \approx 2$. From the previous section an approximating function is given for the real part called x and inserting the expressions for k and introducing $f_1 \approx c/2L$ for the fundamental frequency and the diameter $D = 2a$ the function becomes a constant related to the mechanical construction of the pipe through the parameter L/D.

$$x = \frac{\left(\frac{ka}{2}\right)^2}{\sqrt{1+\left(\frac{ka}{2}\right)^4}} = \frac{\left(\frac{\pi\, fa}{c}\right)^2}{\sqrt{1+\left(\frac{ka}{2}\right)^4}} = \frac{\left(\frac{\pi\, fa}{2\,L\,f_1}\right)^2}{\sqrt{1+\left(\frac{ka}{2}\right)^4}} = \frac{\left(\frac{\pi/4}{L/D}\right)^2\left(\frac{f}{f_1}\right)^2}{\sqrt{1+\left(\frac{ka}{2}\right)^4}}$$

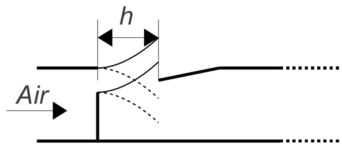
At low frequencies the function simplifies to the nominator showing that narrow pipes are sounding weaker than the wide pipes if the input power remains unchanged.

$$M_C \approx \left(\frac{\pi/4}{L/D}\right)^2\left(\frac{f}{f_1}\right)^2 \quad \text{where} \quad ka < 1.5$$

For the pipe shown with $L/D \approx 27$ the factor is $M_C = 850{\cdot}10^{-6}$ (–61 dB) at the fundamental and the quality factor at resonance was $Q_1 \approx 760$ (58 dB), which combines into –3 dB. The level of the flue oscillation is –6 dB according to the next section so the result becomes –9 dB.

Sound source

An air jet strikes a wedge to produce an edge tone. The cut up height *h* from the air exhaust to the edge of the labium defines the natural flue oscillating frequency as $f_F \approx 0.2\, v/h$ where *v* is the air velocity [3-235]. For *v* = 10 m/s and *h* = 10 mm the frequency becomes 200 Hz and the natural flue frequency of a well-designed pipe approaches the fundamental resonance frequency of the pipe. The oscillation amplitude is enhanced through resonance, which in turn creates feedback to the flue as a frequency-stabilising effect although the pipe may be slightly detuned by changing the air velocity or adjusting details of the cut up.



Appendix-AirJet

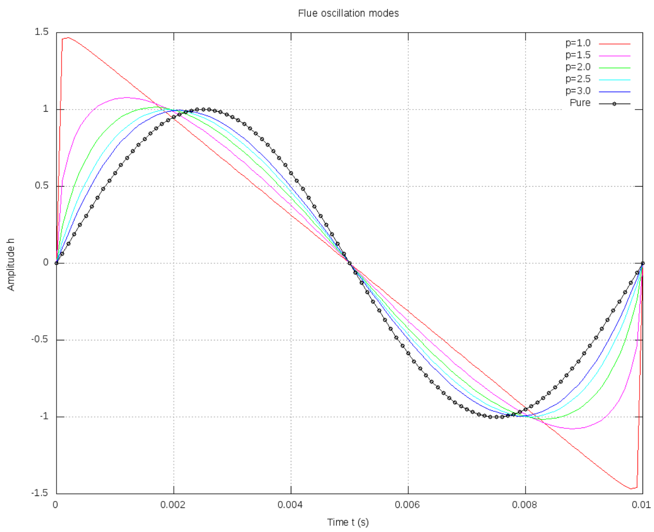
The oscillating air jet, the flue, is assumed to oscillate approximately sinusoidal.

The mechanism behind the flue oscillation is not fully understood according to literature but it is expected related to the mass of oscillating air with the force being due to the air curvature creating a force opposing the movement, which resembles the equation for the pendulum so the oscillation must be fairly smooth. The air pressure within the pipe is increased when the oscillating jet points inward and is decreased when the air jet points outward so almost sinusoidal oscillation is build up within the pipe but the available sources states nothing regarding the non-linearity. The waveform within the pipe is reported as nearly symmetrical [3-238] but since the entire regime of harmonics are present [4-221] some asymmetric must be present, which could be due to an offset within the edge of the labium (as is seen on real pipes) or the flue might oscillate asymmetrical due to the non-linear relation of the ideal gas law.

Examples of periodic waveforms with programmable asymmetry are shown below.

$$h = \sum_{n=1}^{200} \frac{\sin(2\pi n f_1 t)}{n^p}$$

where
$p = 1.0,\ 1.5,\ 2.0,\ 2.5,\ 3.0$



Model of the flue non-linearity by addition of sine waves at harmonically related frequencies (nf_1) and at different relative amplitudes. The waveform for p = 1 is unrealistic so p is larger than unity.

A distortion-free flue oscillation is shown for comparison as the dotted line ($p \to \infty$) while a heavily distorted oscillation is shown as the red line (*p* = 1) with harmonics decaying as 1/*n*. The sawtooth waveform is physically unrealistic since the sudden jump back requires very high acceleration of the mass of the air within the jet; hence $p > 1$. As shown within the previous section the coupling from pipe to environment is proportional to frequency squared so assuming $p \approx 2$ the flue response complements this relation with the result that the output spectrum becomes an almost exact replica of the pipe response, which was actually observed within the previous section.

$$M_F = \left(\frac{f_1}{f}\right)^2 \quad \text{Wide pipe diameter}$$

This applies to wide pipes while the narrow pipes of the string-family show an increased level of the second and third harmonic. This can be modelled using slower initial decay rate ($p \approx 1$) and approaching the second-order relation ($p \approx 2$) at evolved frequency; however, the physical model behind this behaviour is not known.

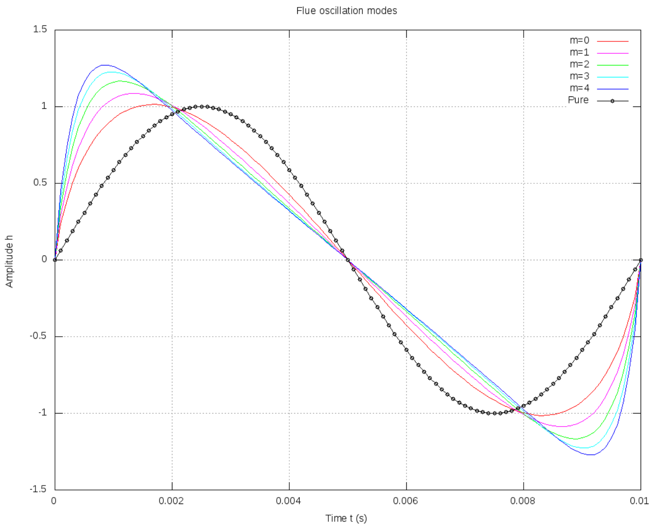
$$h = \sum_{n=1}^{200} \frac{\sin(2\pi n f_1 t)}{n^p}$$

where

$$p = 2 - \frac{1}{\sqrt{1+\left(\frac{n}{m}\right)^2}}$$

and

$$m = 0,\ 1,\ 2,\ 3,\ 4$$



A modified model of the flue oscillation with amplitude approximating 1/n for the lower harmonics and $1/n^2$ for the higher harmonics with m as the division between low and high.

According to the model a cross-over frequency is introduced and is specified through m so the decay of the harmonics is due to integration (ω_1/s) and low-pass filtering with cut-off at $f_m = mf_1$ (i.e. $\omega_m = m\omega_1$) thus defining the frequency for $p \approx 1.4$ of sloping rate. The magnitude function M_F of the model is computed from the Laplace expression using $s = i\omega$.

$$H_F = \frac{\omega_1}{s}\frac{\omega_m}{\omega_m + s} = \frac{\omega_1}{s}\frac{1}{1+s/m\omega_1} \quad \Rightarrow \quad M_F = \frac{f_1}{f}\frac{1}{\sqrt{1+(f/mf_1)^2}}$$

Hence the resulting equation for the pipe spectrum with the resonator response M_R, the oscillation flue response M_F and the response due to the coupling to the environment M_C.

$$M_{RES} = M_F M_R M_C = \left[\left(\frac{\pi/4}{L/D}\right)^2 \frac{1}{\sqrt{1+(f/mf_1)^2}} \frac{f}{f_1}\right] M(f)$$

A relation is needed for m and the timbre parameter L/D will be addressed since it divides between flute, diapason and string family pipes. The value is within the range from 42 to 50 for a standard diapason pipe; less for flute-family pipes and higher for string-family pipes. The value of m should exceed unity to decrease the level of the lower harmonics so the parameter m is <u>defined</u> to equate unity for L/D at the geometric mean of the range for diapason ($L/D \approx 45$), which proves to produce a fairly good simulation result; however, a physical definition is not provided.

$$m \propto \frac{L}{D} \quad \text{and} \quad \frac{L}{D} = \sqrt{42 \cdot 50} = 45 \quad \text{for} \quad m = 1 \quad \Rightarrow \quad m = \frac{1}{45}\frac{L}{D} = \frac{N}{45}$$

For the pipes to be analysed within this document the cut-off parameter becomes $m \approx 0.51$ for the Claribel Flute ($L/D = 23$) so there is virtually constant harmonic decay; $m \approx 1.02$ for the Open Diapason ($L/D = 46$) so the fundamental is attenuated by 3 dB; and finally $m \approx 2.70$ for the Salicional string pipe ($L/D = 122$) where the fundamental is attenuated by 8 dB.

The resulting shape of the pressure oscillation is shown above with parameter $m \approx 0$ (red line) giving $p = 2$, which is identical to the green line within the previous figure.

Scaling

Pipe diameter is not constant throughout the rank of organ pipes since this would produce a very inhomogeneous sound. For a pipe with 50 mm diameter the number of resonant modes is 16 at the bass end producing rich timbre, while there would be only one mode at the treble end with a dull sound as result. Constant ratio of length to diameter results in a fixed number of modes but this produces wrong tonal balance too due to the characteristics of the human ear. The first sixteen harmonics of an 8' bass pipe covers the range from 65 Hz to 1 kHz, which is predominantly low frequencies, while a treble pipe at 8' covers the range from 1 to 15 kHz; thus approaching the limit of hearing so the number of harmonics should be reduced at the high end of the keyboard. A compromise between fixed diameter and fixed length to diameter ratio must be found and it was originally established by trial and error with the diameter changing slower than the length.

Pipe length is given by the fundamental and is halved after 12 semitones, while the optimum range for the pipe width is halving radius after 16 or 17 semitones [3-241]. With an initial radius of a_0 at some key and radius a_n at a distance of n semitones higher the relation becomes as shown with radius being halved after 16 semitones. Pipe radius is thus reduced by 40 % at the octave ($n = 12$).

$$a_n = 2^{-n/16} a_0 \quad \Rightarrow \quad a_{12} = 0.60\, a_0$$

The scaling is in one source[2] given as the double square root of eight, which is $8^{0.25} = 1.68$ and is in fine agreement with the above equation since $2^{12/16} = 1.68$. The same source states that the range for halving the diameter is 17 semitones but the associated table clearly demonstrates that the starting pipe has been counted as well; so the optimum range is 16 semitones.

Using this material the analysed pipes will be transformed to C at 65.4 Hz from the published data measured at at f#1 (370 Hz) for Pykett and c1 (262 Hz) elsewhere. The lengths are scaled to the lower C using the usual 12 semitones for an octave:

$$\begin{array}{ll}
\text{Claribel flute (Pykett):} & L_C = 4\,L_{fs\,1} = 2^{30/12} \cdot (446\,\text{mm}) = 2.523\,\text{m} \\
\text{Open diapason (Pykett):} & L_C = 4\,L_{fs\,1} = 2^{30/12} \cdot (455\,\text{mm}) = 2.574\,\text{m} \\
\text{Open diapason (Douglas):} & L_C = 4\,L_{c\,1} = 2^{24/12} \cdot (645\,\text{mm}) = 2.580\,\text{m} \\
\text{Open diapason (Borner):} & L_C = 4\,L_{c\,1} = 2^{24/12} \cdot (649\,\text{mm}) = 2.596\,\text{m} \\
\text{Salicional (Borner):} & L_C = 4\,L_{c\,1} = 2^{24/12} \cdot (651\,\text{mm}) = 2.604\,\text{m}
\end{array}$$

Radius is scaled using 16 semitones for doubling:

2 See http://en.wikipedia.org/wiki/Organ_flue_pipe_scaling.

$$
\begin{array}{ll}
\text{Claribel flute (Pykett):} & a_C = 2^{30/16}\cdot(20\,\text{mm}) = 73.4\,\text{mm} \\
\text{Open diapason (Pykett):} & a_C = 2^{30/16}\cdot(8.3\,\text{mm}) = 30.4\,\text{mm} \\
\text{Open diapason (Douglas):} & a_C = 2^{24/16}\cdot(11.0\,\text{mm}) = 31.1\,\text{mm} \\
\text{Open diapason (Borner):} & a_C = 2^{24/16}\cdot(5.9\,\text{mm}) = 16.7\,\text{mm} \\
\text{Salicional (Borner):} & a_C = 2^{24/16}\cdot(4.0\,\text{mm}) = 11.3\,\text{mm}
\end{array}
$$

The length to diameter ratio becomes as follows where the range 42 to 50 designates a diapason flute; hence, the Claribel flute is indeed of flute-family while the open diapason according to Borner should be regarded as a string-family pipe along with the Salicional pipe.

$$
\begin{array}{ll}
\text{Claribel flute (Pykett):} & L/D = 2523/(2\cdot 73.4) = 17.2 \\
\text{Open diapason (Pykett):} & L/D = 2574/(2\cdot 30.4) = 42.3 \\
\text{Open diapason (Douglas):} & L/D = 2580/(2\cdot 31.1) = 41.5 \\
\text{Open diapason (Borner):} & L/D = 2596/(2\cdot 16.7) = 77.7 \\
\text{Salicional (Borner):} & L/D = 2604/(2\cdot 11.3) = 115.2
\end{array}
$$

Resultant pipe spectrum

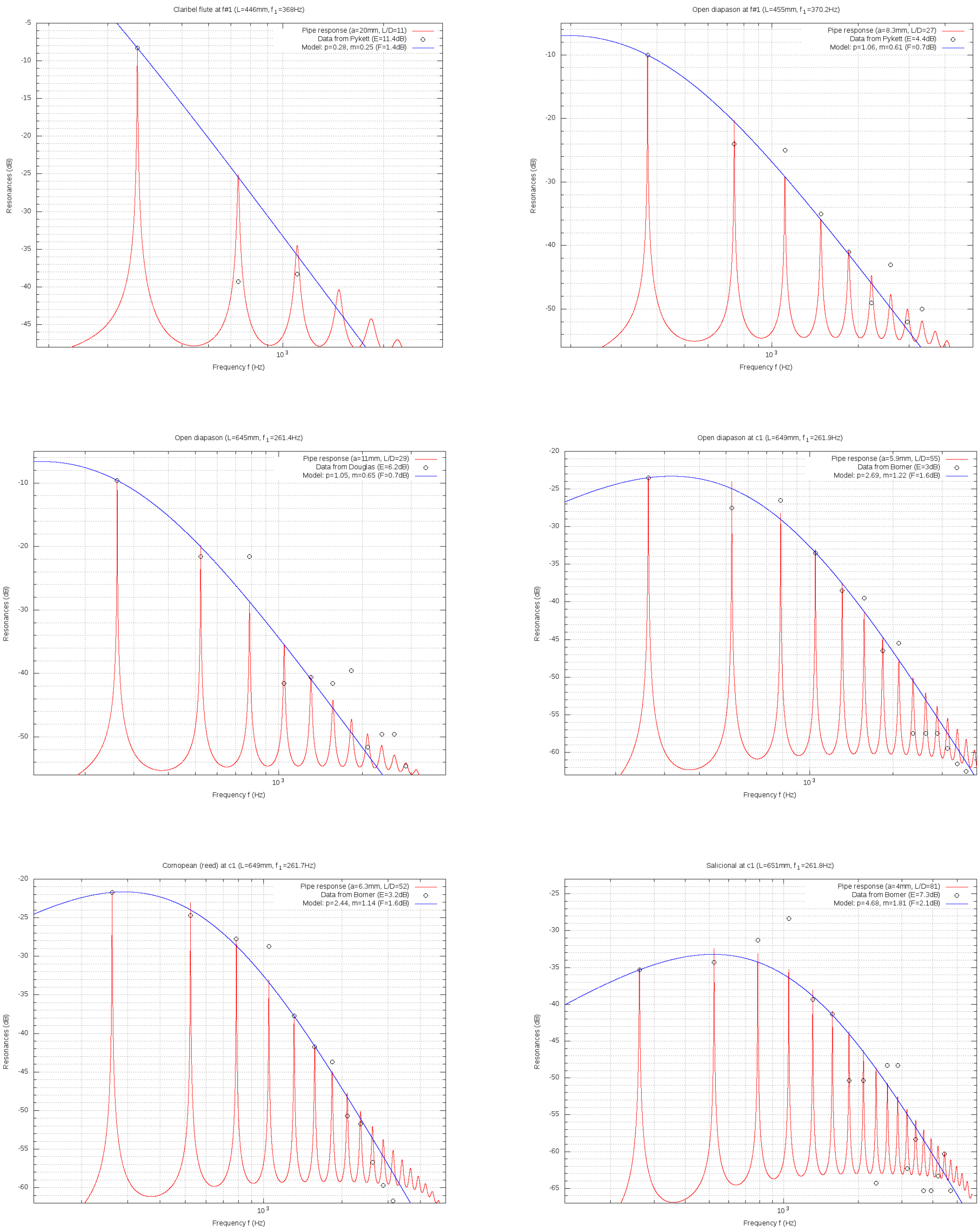
Assembling the findings from the previous sections the pipe spectrum becomes as follows where the resonator modes are assumed integral multiples of the fundamental frequency, which is not strictly true but the higher modes are broadened by the increased damping so the effect of modes being offset from the flue harmonics is not assumed significant.

$$
M(f) = \frac{\left(\frac{\pi/4}{L/D}\right)^2}{\sqrt{1+\left(\frac{f}{mf_1}\right)^2}} \frac{f}{f_1} \sum_{n=1}^{N} \left| \frac{f_n f_1}{(1+\delta_n^2) f_n^2 - f^2 + i\,2\,\delta_n f_n f} \right| \qquad
\begin{array}{c}
f_n = n f_1 \\
f_1 \approx c/(2L+D) \\
N \approx L/D \\
m \approx N/45 \\
\delta_n = \delta_R + \delta_V + \delta_A
\end{array}
$$

A simulation run of the acoustical output from organ pipes is shown below for six pipes sounding at c1 (262 Hz) or f#1 (370 Hz); which is midway through the keyboard with four or five octave range. The red line is the simulated response with measurement data as the black circles and the blue line is an approximating filter to be described later. There are basically only two input parameters to the simulation: the physical length from labium to the open end and radius of the pipe so the timbre is to a large degree dependent upon these parameters only. But note that this represents a model and that the actual sound of the pipe dependent upon more than just the correct spectrum (see the note on the first page).

The simulation is checked against reported spectra and the error is reported as the root mean square of the difference between the response *M*(*f*) and pipe data *P*(*f*) using the lower half of the harmonics and progressively attenuating to focus on the correctness of the lower harmonics.

$$
E = \sqrt{\sum_{n=1}^{N/2} \frac{(M(f_n) - P(f_n))^2}{n}}
$$



The amplitude response for six pipes shown as the red line with measurements as black circles and an approximating filter in blue. The filter is controlled through parameter p being responsible for the low-pass filter cut-off (third order) and parameter m for defining the high-pass filter cut-off. Note that Cornopean is a reed pipe so the simulation does not apply in the strict sense; however, the approximation is fair. Note also that the increasing level of the third and fourth harmonic for the Salicional pipe is not correctly modelled despite the inclusion of the m parameter.

Approximating filter

The blue line represents an attempt defining the filter circuit required for pipe emulation within an electronic organ, and is based upon the generic first-order low-pass filter $H_{LP}(f)$ shown below with the Laplace operator substituted by $i\omega$ where $\omega = 2\pi f$. The pole at f_P is related to the fundamental frequency of the pipe as $f_P = pf_1$ where p is a parameter defined from the simulated spectrum. The filters $H_2(f)$ and $H_3(f)$ use equivalent expressions with poles p_2 and p_3.

$$H_{LP}(f) = \frac{\omega_p}{\omega_p + i\omega} = \frac{p f_1}{pf_1 + if} = \frac{1}{1 + if/pf_1} \quad \Rightarrow \quad H_1(f) = \frac{1 + i/p_1}{1 + if/p_1 f_1}$$

Three such filters are combined into a third-order passive low-pass filter and all filter sections are scaled for unity gain at the fundamental frequency $f = f_1$ regardless of the value of the parameters thus easing modelling since it aligns the low-pass filter with the peak of the simulated spectrum for the pipe at the fundamental frequency. The assumption of using a passive filter implies that the resultant response is defined through independent mechanisms.

The parameters were initially found through trial and error for best fit to the simulated response and fair approximation was found using poles at the same frequency for all filters: $p = p_1 = p_2 = p_3$. An algorithm was generated to derive the parameter directly from the simulated spectrum removing the time-consuming manual curve fitting procedure. The resulting filter is 9 dB down at the cut-off frequency, 21 dB down at the octave where the slope has increased to –12 dB/octave and the slope approaches –18 dB/octave at higher frequencies fitting nicely to the average pipe spectrum.

An additional filter was added after some experiments to increase the level of the second and third harmonics relative to the fundamental for emulation of the narrow-width string-family pipes and this proved to be related to the L/D parameter as defined within the previous section.

The response becomes:

$$M(f) = \left(\frac{1}{\sqrt{1 + (f/pf_1)^2}}\right)^3 \frac{f/mf_1}{\sqrt{1 + (f/mf_1)^2}} \quad \text{where} \quad m = \frac{L/D}{45}$$

Since the correction due to L/D is known at this point within the analysis the expression of the filter response will be written as a passive third-order low-pass filter hereafter called $A(f)$.

$$A(f) = M(f)\left(\frac{f/mf_1}{\sqrt{1 + (f/mf_1)^2}}\right)^{-1} = \left(\frac{1}{\sqrt{1 + (f/f_C)^2}}\right)^3 \quad \text{where} \quad f_C = pf_1$$

The cut-off frequency f_C is is found using the third root and squaring (i.e. raising to the power 2/3), then rearranging and finally using the square root.

$$A^{2/3}(f) = \frac{1}{1 + (f/f_C)^2} \quad \Rightarrow \quad \frac{f}{f_C} = \sqrt{\frac{1}{A^{2/3}(f)} - 1} \quad \Rightarrow \quad f_C = \frac{f}{\sqrt{A^{-2/3}(f) - 1}}$$

The available input frequencies are the response peaks at $f = nf_1$ so the magnitude function $A(f)$ is known for $A_n = A(nf_1)$ where the notation $M_{max}(n) = M(nf_1)$ is used within the software. The value of $M_{max}(n)$ is normalised by division with $\alpha M_{max}(1)$ since $M_{max}(1)$ compensates for the scaling due to the quality factor within the expression for the resonator response and $\alpha > 1$ guarantees $M_{max}(1) < 1$ since the amplitude must decay monotonously starting from the level of the horizontal asymptote.

$$f_C = \frac{nf_1}{\sqrt{A_n^{-2/3} - 1}} \quad \text{where} \quad A_n = \frac{M_{max}(n)}{\alpha M_{max}(1)} \quad \text{and} \quad M_{max}(n) = \left(\frac{1}{\sqrt{1 + (n/3)^2}}\right)^3 \frac{n/m}{\sqrt{1 + (n/m)^2}}$$

Parameter α is defined using the first two values A_1 and A_2 from the simulated spectrum by the requirement of equal cut-off frequencies for both amplitude values.

$$f_C = \frac{f_1}{\sqrt{A_1^{-2/3}-1}} = \frac{2 f_1}{\sqrt{A_2^{-2/3}-1}} \quad \Rightarrow \quad \sqrt{A_2^{-2/3}-1} = 2\sqrt{A_1^{-2/3}-1} \quad \Rightarrow \quad 4 A_1^{-2/3} - A_2^{-2/3} = 3$$

Using the definition of A_1 and A_2 gives an equation in α to be solved. The negative sign within the exponent is removed by inversion so the first term becomes $4\alpha^{2/3}$. The parameter is isolated and the required relation is found.

$$4\alpha^{2/3} - \left(\frac{M_{max}(2)}{\alpha M_{max}(1)}\right)^{-2/3} = 3 \quad \Rightarrow \quad \alpha^{2/3}\left(4 - \left(\frac{M_{max}(1)}{M_{max}(2)}\right)^{2/3}\right) = 3 \quad \Rightarrow \quad \alpha = \left(\frac{3}{4 - \left(\frac{M_{max}(1)}{M_{max}(2)}\right)^{2/3}}\right)^{3/2}$$

The cut-off frequency f_C is then found as the average value for $n = 1, \dots N$, where N is the number of reported data points from the references of pipe spectra. The number of points N is calculated using either L/D or the actual number of data points within the reported spectrum, whichever is the lowest to avoid index overrun.

Conclusion

Using the reported spectra parameter the range for α is from 1 to 3 with most of the pipes and the values of α and m for the *Claribel flute* illustrates that this pipe is effectively integrated three times into sine-wave shape.

Key parameters for selected pipes ranging from flute to string pipes. Note that the calculated value for L/D is for the shown key and not the lowest C key on the keyboard.

Pipe	*Key*	*L/D*	*α*	*p*	*m*
Claribel flute (Pykett)	f#1	11	57	0,28	0,25
Open diapason (Pykett)	f#1	27	2,54	1,06	0,61
Open diapason (Douglas)	c1	29	2,60	1,05	0,65
Open diapason (Borner)	c1	55	1,16	2,69	1,22
Salicional (Borner)	c1	81	1,04	4,68	1,84

According to the data for the *Open diapason* (Pykett) the cut-off frequency of the filter is almost right at the fundamental ($2^{1/12}$ = 1.06 is one semitone higher) while filter cut-off for the *Salicional* string pipe is more than two octaves above ($2^{27/12}$ = 4.76 is almost 27 semitones higher). But this is not the only result from the emulation. The filters for the flute-family and the wide diapason pipes (the three first simulations) show a second harmonic *attenuated* by 10 to 15 dB compared to the fundamental while the Salicional string pipe shows the level of the lower harmonics being *increased* 3 dB compared to the fundamental.

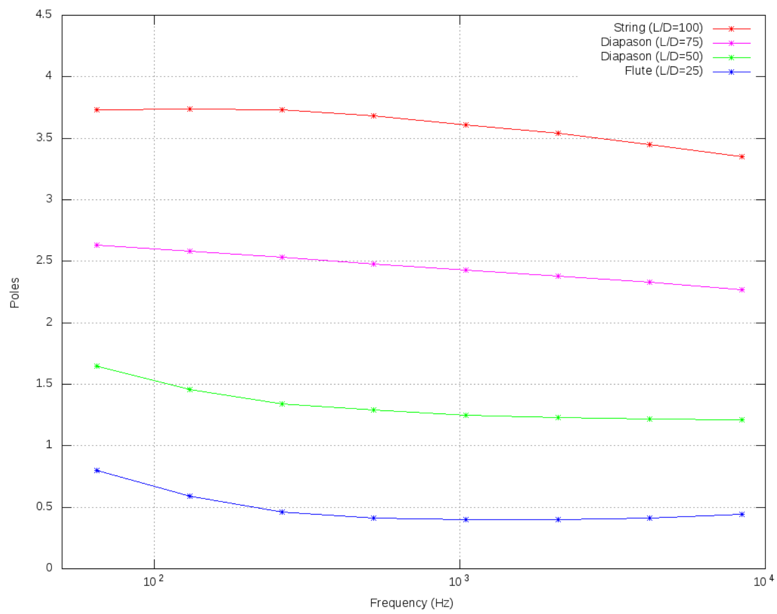
The required set of parameters are shown below for the different pipe categories represented through the parameter values L/D = 25, 50, 75 and 100. The simulation runs used a calculated length to produce the correct fundamental frequency and radius was set to generate the required L/D ratio at the bottom C key of 65.4 Hz.

The simulated response generates a value for the filter pole parameter p with the value shown in the table below, while parameter m is given directly by the L/D ratio as is defined in the previous section.

The parameters for four pipe types with length to diameter ratio specified at C (65.4 Hz) and calculated for the higher pitches. The remaining parameter is m = (L/D)/45.

Pipe	Interpolating filters				Flute		Diapason				String	
	8'	***4'***	***2'***	***1'***	***L/D***	***p***	***L/D***	***p***	***L/D***	***p***	***L/D***	***p***
c6 (8371.2 Hz)				TOP	7	0,44	13	1,21	20	2,27	27	3,35
c5 (4185.6 Hz)			TOP	↑	8	0,41	16	1,22	25	2,33	33	3,45
c4 (2092.8 Hz)		TOP	↑	MID	10	0,40	20	1,23	30	2,38	40	3,54
c3 (1046.4 Hz)	TOP	↑	MID	↑	12	0,40	24	1,25	36	2,43	48	3,61
c2 (523.2 Hz)	↑	MID	↑	BOT	14	0,41	28	1,29	43	2,48	58	3,68
c1 (261.6 Hz)	MID	↑	BOT		17	0,46	34	1,34	52	2,53	69	3,73
c (130.8 Hz)	↑	BOT			21	0,59	41	1,46	62	2,58	83	3,74
C (65.4 Hz)	BOT				25	0,80	50	1,65	75	2,63	100	3,73

The table values are plotted below and the curves are symmetrical around *L*/*D* = 75, probably due to the equations for the damping constants but this issue has not been investigated further. It is somewhat surprising that the change within the slope of the cut-off frequency through the keyboard range is minute; however, this is not a scientific proof of the resulting pipe response.



Plot of the table values showing the relative variation. The dots represents the C-keys ranging from C (65 Hz) to c6 (8380 Hz).

Note that the parameter *m* is given by the *L*/*D* ratio and does not need to be plotted.

Software

The document is based upon the following code, which is initiated by selecting the parameters related to the pipe at hand: the pipe length *L* and radius *a* = *D*/2 where *D* is the pipe diameter. The first selections are reported spectra on organ pipes with the source recorded and the final selection is model pipes at different pitch and with *L*/*D* as the parameter. PipeData have been calculated for the experimental pipes (the “otherwise” selection) using an empiric equation since plotting requires data to be present. The remaining material is covered within the article.

```
% Organ pipe response.
clear
% --------------------------------------- MEASUREMENT DATA
% 1: Open diapason, Douglas
% 2: Open diapason, Borner
% 3: Salicional, Borner
% 4: Cornopean (reed), Borner
% 5: Organ reed, Borner
% 6: Claribel flute, http://www.pykett.org.uk/how_the_flue_pipe_speaks.htm
% 7: Open diapason, http://www.pykett.org.uk/how_the_flue_pipe_speaks.htm
% 8: Experiment
Select='3';                              % Sequence 6 7 1 2 4 3
switch Select
case {'1'}
    PipeName='Open diapason'; SourceName='Douglas';
    PipeData=[60 48 48 28 29 28 30 18 20 20 15 11 8 1 0 2];
    L=0.645; a=0.011; %a=0.011;
case {'2'}
    PipeName='Open diapason at c1'; SourceName='Borner';
    PipeData=[60 56 57 50 45 44 37 38 26 26 26 24 22 21 18 17];
    L=0.649; a=0.0059; %a=0.0063;
case {'3'}
    PipeName='Salicional at c1'; SourceName='Borner';
    PipeData=[53 54 57 60 49 47 38 38 24 40 40 26 30 23 23 25 28 23 17 16 14 12 ];
    L=0.651; a=0.0040;% a=0.0042;
case {'4'}
    PipeName='Cornopean (reed) at c1'; SourceName='Borner';
    PipeData=[60 57 54 53 44 40 38 31 30 25 22 20 14 10 14 6 7 5 7 ];
    L=0.649; a=0.0063; %a=0.0070;
case {'5'}
    PipeName='Organ reed at c1'; SourceName='Borner';
    PipeData=[36 38 40 39 38 39 37 38 36 35 32 32 22 20 18 18 19 19 18 17 17 17 16
16 16 14 6 16 6 5 4 4 5 4 3 3 3 2 2 4 4 3 3];
    L=0.654; a=0.0030; b=0.02;
case {'6'}
    PipeName='Claribel flute at f#1'; SourceName='Pykett';
    PipeData=[60 29 30 18 19 12 10 6];
    L=0.446; a=0.020; %a=0.015; b=-0.0;
case {'7'}
    PipeName='Open diapason at f#1'; SourceName='Pykett';
    PipeData=[60 46 45 35 29 21 27 18 20 12 14 8 6];
    L=0.455; a=0.0083; %a=0.0086;
% --------------------------------------- EXPERIMENTAL PIPE
otherwise
   PipeModel='LD75'                        % Define the range to plot.
    switch PipeModel
        case {'LD25'} a0=0.0185; b=0.0; PipeData=-10*log10(2+(1:8).^5.8);
        case {'LD50'} a0=0.0094; b=0.0; PipeData=-10*log10(4+(1:10).^4.7);
        case {'LD75'} a0=0.00625; b=0.0; PipeData=-10*log10(15+(1:10).^4.5);
        case {'LD100'} a0=0.0047; b=0.0; PipeData=-10*log10(28+(1:20).^3.9);
    end
    Pitch='c1';                           % Select the pipe size.
    PipeName=['Experiment at ' Pitch]; SourceName='Experiment';
    switch Pitch
```

```
        case {'c6'} s=128; a=a0*0.6^5;
        case {'c5'} s=64; a=a0*0.6^4;
        case {'c4'} s=32; a=a0*0.6^3;
        case {'c3'} s=16; a=a0*0.6^2;
        case {'c2'} s=8; a=a0*0.6;
        case {'c1'} s=4; a=a0;
        case {'c'}   s=2; a=a0/0.6;
        case {'C'}   s=1; a=a0/0.6^2;
    endswitch
    L=343/(2*s*65.4)-1.2*a;
endswitch
% ---------------------------------------- INPUT PARAMETERS
LD=L/(2*a);                               % Pipe length to diameter (-).
c=343;                                    % Speed of sound (m/s).
rho=1.2;                                  % Mass density of air (kg/m3).
eta=1.8e-5;                               % Viscosity of air (kg/sm).
h=a/2;                                    % Labium height (m).
w=a;                                      % Labium width (m).
f1=c/(2*L+2*a);                           % Pipe resonance (Hz).
f0=30;                                    % Frekvens ved start (Hz).
df=f1/4000;                               % Frekvens mellem trin (Hz).
N=max(10, round(L/(2*a)));                % Highest mode number is shown, but
N=min(N,length(PipeData));                % not less than the available data.
Fmax=500*round(N*f1/500);                 % Upper freq limit within plot.
f=f0:df:N*f1;                             % Frequency range (Hz).
ka=2*pi*f*a/c;                            % Wavenumber for reflection R1 (1/m).
kb=sqrt(w*h/pi)*ka;                       % Wavenumber for reflection R2 (1/m).
disptxt=[PipeName ' (' SourceName '): f1='  num2str(f1) 'Hz ' ...
                 'a=' num2str(1000*a) 'mm N=' num2str(N)];
disp(disptxt)
% ---------------------------------------- REFLECTION FROM PIPE END
function R0=reflection (kr)
    RM=((kr/2).^2)./sqrt(1+(kr/2).^4);    % Radiation resistance.
    XM=(kr/1.7)./sqrt(1+(kr/1.7).^6);     % Radiation reactance.
    ZM=RM+i*XM;                           % Radiation impedance.
    R0=(ZM-1)./(ZM+1);                    % Reflection constant.
end
% ---------------------------------------- DAMPING PARAMETERS
R1=reflection(ka);                        % Reflection at open end.
R2=reflection(kb);                        % Reflection at mouth.
dR=-log(abs(R1).*abs(R2))/(2*pi);         % Resulting reflection coefficient.
dV=2*eta./(pi*rho*f*a^2);                 % Viscosity losses.
m=0.64e-3+(0.31e-3)*(f/1000).^2;          % Air attenuation factor (1/m).
dA=m*L/(2*pi);                            % Air attenuation losses.
% ---------------------------------------- DAMPING COEFFICIENT
figure(1)
dn=dR+dV+dA;                              % Resultant attenuation.
TitleText=[PipeName ' (L=' num2str(round(1000*L)) ...
          'mm, f_1=' num2str(round(10*f1)/10) 'Hz)'];
semilogy(f,dn,'-r', f,dR,'-g', f,dV,'-b', f,dA,'-m')
grid on
title(TitleText)
xlabel('Frequency f (Hz)')
ylabel('Damping')
legend('Resultant damping','Reflection damping','Viscous damping', ...
          'Air absorption','Location','East')
axis([0 Fmax/2 1e-4 1e-2])
%print -dpng /home/tore/Dokumenter/ElectronicOrgan/printfile.png
% ---------------------------------------- PIPE RESPONSE
figure(2)
m=LD/45;                                  % Cross-over frequency.
MR=0;                                     % Initialise resonator.
MC=((pi/4)/LD)^2;                         % Scale factor for coupling.
for n=1:N
    fn=n*f1;                              % Mode frequency.
```

```
    M1=(f1./f)./sqrt(1+(f/(m*f1)).^2);                      % Flue response.
    M2=abs(f1*fn./((1+dn.^2)*fn^2-f.^2+i*2*dn.*f*fn));    % Resonator response.
    M3=MC*(f/f1).^2;                                        % Coupling response.
    Mn=M1.*M2.*M3;                        % Response at harmonic n.
    [Mmax(n),ix(n)]=max(Mn);              % Get maximum value and index to maximum.
    A(n)=20*log10(Mmax(n));               % Store peak value (dB).
    MR=MR+Mn;                             % Combine the terms.
end
MR=20*log10(abs(MR));                     % Amplitude spectrum (dB).
MRmax=5*ceil(max(MR/5));                  % Get maximum of spectrum (dB).
Pmax=max(PipeData);                       % Get maximum of pipe data (dB).
if (Pmax>PipeData(1))                     % Set maximum for plot.
    MRmax=MRmax+Pmax-PipeData(1);
end
ix0=round((ix(1)+ix(2))/2);               % Get minimum value between harm 1 and 2.
%MRmin=5*floor(MR(ix0)/5);                 % Set minimum for plot (dB).
MRmin=floor(MR(ix0));                     % Set minimum for plot (dB).
% ---------------------------------------- MARKERS
PD=NaN*ones(1,length(MR));                % Reserve space for the vector,
P1=A(1)-PipeData(1);                      % Fit markers to f1.
for n=1:N                                 % Work through N modes.
    PD(ix(n))=PipeData(n)+P1;             % Use table values for the mark.
end
% ---------------------------------------- FILTER APPROXIMATION ERROR
E=0;                                      % Deviation between simulation and
markers.
for n=1:round(N/2)                                 % Work through N modes.
    E=E+((MR(ix(n))-PipeData(n)-P1)^2)/n; % Mean square error of simulation.
end
E=sqrt(E);                                % RMS of spectrum simulation error (dB).
% ---------------------------------------- CUT-OFF FREQUENCY
for n=1:N
    beta(n)=m*sqrt((1+(n/m)^2))/n;      % Amplitude correction.
end
alpha=(3/(4-(Mmax(1)*beta(1)/(Mmax(2)*beta(2)))^(2/3)) )^(3/2);
for n=1:N
    Mx=Mmax(n)*beta(n)/(alpha*Mmax(1)*beta(1));
    CutOff(n)=n*f1/sqrt(1/Mx^(2/3)-1);
end
CutOff(1)=CutOff(2);
fc=mean(CutOff);
p3=round(100*fc/f1)/100;
% ---------------------------------------- APPROXIMATING FILTER
H1=(1+i/p3)./(1+i*f/(p3*f1));             % One filter section (compensated).
H2=(f/(m*f1))./sqrt(1+(f/(m*f1)).^2);     % Narrow pipe family correction.
H3=A(1)+20*log10(abs((H1.^3).*H2*beta(1)));
F=0;                                      % Deviation between simulation and
filter.
for n=1:N
    F=F+((H3(ix(n))-A(n))^2)/n;           % Mean square error of filter.
end
F=sqrt(F);                                % RMS of filter approximation error (dB).
disptxt=['N=' num2str(N) ', alpha=' num2str(alpha) ', fC=' num2str(fc) ...
         ', E=' num2str(E) 'dB, F=' num2str(F) 'dB'];
disp(disptxt)
% ---------------------------------------- PLOT RESPONSES
semilogx(f,MR,'-r', f,PD,'ok','markersize',10, f,H3,'-b')
grid minor on
title(TitleText)
xlabel('Frequency f (Hz)')
ylabel('Resonances (dB)')
axis([f1/2 Fmax MRmin MRmax])
legend(['Pipe response (a=' num2str(round(1e5*a)/100) 'mm, L/D=' num2str(round(LD))
')'], ...
            ['Data from ' SourceName ' (E=' num2str(round(10*E)/10) 'dB)'] , ...
```

```
                ['Model: p=' num2str(round(100*p3)/100) ...
                 ', m=' num2str(round(100*m)/100) ' (F=' num2str(round(10*F)/10)
'dB)'])
print -dpng /home/tore/Dokumenter/ElectronicOrgan/printfile.png
%print -dpng /Users/tas/Documents/ElectronicOrgan/printfile.png
```

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