Report 2

Absorption

Written by

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Abstract

This document is part of a series of reports covering the topic "Room Acoustics", which was conducted as a self study course with kind assistance from Jonas Brunskog from the Danish Technical University.

The objective of the report is absorption. The absorption coefficient is defined from the acoustical impedance of the surface, so the study concentrates on determining the surface impedance on selected applications, including (1) the membrane absorber, which may represent a large window or a protective cover for vulnerable absorption materials, (2) the porous absorber using mineral wool in front of a rigid surface, (3) fabric hung in front of a rigid wall, and (4) the Helmholtz resonator.

1. Sound absorption

Summary – Reverberation is introduced as a motivation for estimation of the absorption coefficient for different kind of absorbers. It is shown that an expression for the reflection coefficient is needed and leads to the analysis of acoustical impedance. An iterative method is introduced for determining the surface impedance of a sequence of acoustical elements thus leading to a practical tool for use within acoustical simulation. The resistance to air flow is lightly introduced and the properties of the absorbing material are described according to the model of Delany and Bazley.

1.1. Reverberation

One of the most important room parameters is the reverberation time, which is the duration for the sound energy to decay to one millionth of the initial level. Sabine's equation was published around 1890 and is the de facto equation for reverberation estimation. It uses the volume V of the room and the equivalent absorption area A, which is the sum of surface sections with area S_i and absorption coefficient α_i. The factor $55.3/c$ equals 0.161 using SI-units with c = 343 m/s for the speed of sound. The second equation is due to Eyring and was published 1930 as an improvement for rooms with high levels of absorption where Sabine's equation predicts too high levels of reverberation. Air absorption is shown below for Eyring's equation ($m \approx 10^{-6}\ \mathrm{m}^{-1}$ at 1 kHz) but is often ignored.

$$T_{60} = \frac{55.3}{c}\frac{V}{A} \qquad A = \sum_i \alpha_i S_i \qquad \text{Sabine}$$

$$T_{60} = \frac{55.3}{c}\frac{V}{4mV - S\ln\left(1-\bar{\alpha}\right)} \qquad \bar{\alpha} = \frac{1}{S}\sum_i \alpha_i S_i \qquad \text{Eyring}$$

Published values of the absorption coefficient are measured either within an impedance tube or within a reverberant room. In the former case the measurement is limited to plane waves at normal incidence so random incidence is not studied. The latter case is complicated by edge diffraction at the edges of the sample, which is bending the sound wave around the corner thus increasing the effective absorption area and leading to greater absorption than expected from the physical area. The published figures may sometimes exceed unity due to the measurement uncertainties[1].

Limitations

The relation to the physical position of the absorption material within the room and the shape of the room are not reflected within the equations. The modes or resonance frequencies may decay at different rates if the absorption material is placed in such a way that only some of the modes are affected; this could be the result of adding the major part of the absorption material to one surface only, and results in fast decay slope while the affected modes decay followed by a gradual change into a slower rate while the remaining modes decay. Another issue is the importance of early reflections, which is responsible for the impression of direction and loudness and is important for speech intelligence, but this too is not addressed here.

1.2. Absorption coefficient

Energy is lost when sound propagates in small spaces, such as the interconnected pores of a porous absorber or narrow tubes, and is primarily due to viscous boundary layer effects, although losses due to thermal conduction from air into the absorber may become significant at low frequencies. Typical porous absorption materials include carpets, curtains, cushions, cotton, open cell foam and mineral wools such as fibreglass [CA-156].

[1] See for instance: http://www.leoberanek.com/pages/sabineandeyringeq.pdf.

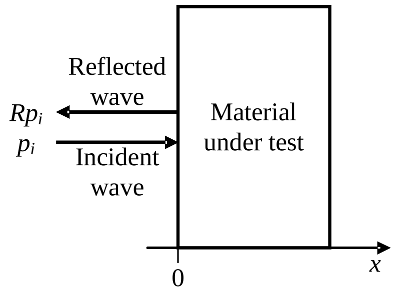
The absorption coefficient is the ratio between the absorbed intensity and the total intensity of the incident wave. The figure is real since the definition uses intensity (i.e. power) so there is no phase information. The absorbed sound wave is calculated using the reflection coefficient. The intensity of the incident wave is $I_i = p_0^2/\rho_0 c$ and the intensity of the reflected wave is $I_r = |R|^2 p_0^2/\rho_0 c$, where R is the complex reflection coefficient and the magnitude is used since the intensity must be real. The intensity of the absorbed wave I_a becomes the difference between the incident intensity and the reflected intensity.

$$\alpha = \frac{I_a}{I_i} = \frac{I_i - I_r}{I_i} = 1 - \frac{I_r}{I_i} = 1 - \frac{|R|^2 p_0^2}{p_0^2} \quad \Rightarrow \quad \boxed{\alpha = 1 - |R|^2}$$

The absorption coefficient is maximum ($\alpha = 1$) when there is no reflection from the surface ($R = 0$), so all energy enters the sample and is dissipated, and minimum ($\alpha = 0$) when there is full reflection ($R = 1$ or $R = -1$), which means that no energy enters the sample.

1.2.1. Reflection coefficient

A sound wave is reflected from a hard surface such as a concrete wall while a soft material absorbs the sound and converts the energy to heat through friction losses or transmission into another room. The characteristic impedance of air is $\rho_0 c \approx 410$ kg m^{-2} s^{-1} at 20ºC and a surface approaching this impedance will not reflect the incident energy so the absorption coefficient is unity. The value of the surface impedance is thus needed for the determination of the reflection coefficient.



The incident wave is attenuated and optionally phase shifted, which is represented by the complex reflection coefficient.

The solution to the wave equation consists of the superposition of two plane waves, one travelling along the positive direction of the x-axis and the other travelling in the negative direction. The latter wave is due to the reflection at the surface and has smaller amplitude and the phase is changed, and both are described by the complex reflection coefficient R [K-36].

$$p(x) = p_i \exp(i\omega t - ikx) + Rp_i \exp(i\omega t + ikx), \quad k = \frac{\omega}{c}$$

The objective is the determination of the reflection coefficient and this starts by introducing the particle velocity from the definition of conservation of momentum [R1-4].

$$\frac{\partial p}{\partial x} = -\rho_0 \frac{\partial v}{\partial t} = \quad \Rightarrow \quad \frac{\partial p}{\partial x} = -i\omega\rho_0 v \quad \Rightarrow \quad v = \frac{1}{-i\omega\rho_0}\frac{\partial p}{\partial x}$$

Differentiating and using $k = \omega/c$ the expression for the particle velocity becomes:

$$v(x) = \frac{p_i}{\rho_0 c}\exp(i\omega t - ikx) - \frac{Rp_i}{\rho_0 c}\exp(i\omega t + ikx)$$

At the surface of the material ($x = 0$) we get:

$$p(0) = (1 + R)\, p_i \exp(i\omega t)$$

$$v(0) = (1 - R)\, \frac{p_i}{\rho_0 c} \exp(i\omega t)$$

The surface impedance is given by the quotient between sound pressure and particle velocity.

$$Z_S = \frac{p(0)}{v(0)} = \rho_0 c\, \frac{1 + R}{1 - R}$$

Solving for the reflection coefficient we get $R = 1$ for the hard wall where $Z_S \to \infty$, which produces a sound pressure at the surface of the wall of $p(0) = 2p_i$ so the sound pressure is doubled at the wall. The coefficient becomes $R = 0$ when the surface impedance matches the characteristic impedance of air $Z_S = \rho_0 c$, which is the goal for the anechoic room and is exemplified through an open window. The coefficient becomes $R = -1$ for a soft wall with $Z_S = 0$, which is approximated within a musical instrument as the open end of the resonator and is the reason for the usefulness as a resonator[2].

$$\boxed{R = \frac{Z_S - \rho_0 c}{Z_S + \rho_0 c}}$$

Using the equation within section 1.2 the absorption coefficient can be estimated.

1.2.2. Local reacting surface

The propagation direction within many porous absorbers is approximately normal to the surface, even for oblique incidence sound. This is due to refraction[3] and it means that the reaction of the material at any point is (almost) independent of the reaction at other points. The surface is termed *locally reacting*, and this is an extremely useful first order approximation [CA-189].

Local reaction at the surface applies if the normal component of the particle velocity at any surface element depends only on the sound pressure at that element and not on the pressure at neighbouring elements. This kind of surfaces is the exception; they are encountered whenever the wall itself or the space behind it is unable to propagate waves or vibrations in a direction parallel to its surface. This is not true for a panel whose neighbouring elements are coupled together by bending stiffness. Moreover this does not apply to a porous layer with an air space between it and a rigid rear wall, unless rigid partitions are used to obstruct the air space and prevent sound propagation parallel to the surface [K-44].

Comparing to normal incidence the absorption coefficient is reduced at random incidence and the frequency of peaks and dips is altered [CA-225]. A locally reacting wall cannot be totally absorbent to random incidence; the theoretical upper limit is $\alpha = 0.95$ for $Z_S = 1.57\rho_0 c$ [K-55].

It is assumed that the incident wave is plane and normal to the surface. The estimation accuracy is thus compromised but the main objective of this report is not to achieve scientific accuracy but to study the behaviour of selected acoustical setups in order to acquire the theory.

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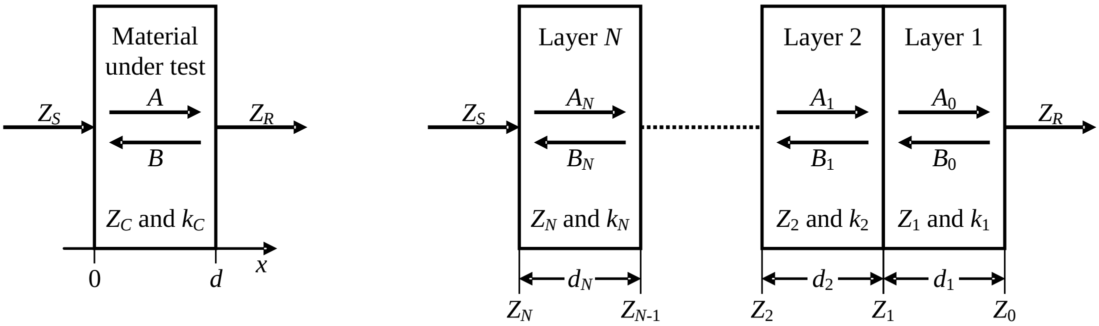
[2] The reflected sound is inverted so the sound pressure at the opening is $p(0) \approx 0$ and almost all of the incident sound is reflected into the resonator. If the instrument is open at the other end as well, standing waves are possible at frequencies where the length of the resonator equals half wavelength; i.e. $f = c/2L$.

[3] The speed of sound within the absorption material is slower than within air so the sound wave is bending at the surface and the angle of incidence is reduced so the wave within the absorption material is approximately normal to the surface.

1.3. Iterative method

Sound is reflected at the boundary between layers of different acoustical properties so the surface impedance of one layer is the rear side impedance of the following layer and so forth. An iterative method is based upon this observation and shall be presented below.



The impedance at the surface Z_S depends not only on the absorbent material, defined through Z_M and k_M, and its thickness d but also on the terminating impedance Z_R at the rear side. The method can be generalised to any sequence of different layers.

The derivation starts with one layer and is generalised into any sequence of layers with different acoustical properties. The layer is described by the characteristic impedance Z_C, the angular wave number k_C and the thickness d. The sound pressure of the travelling waves within the material is defined from their amplitudes A and B. These two parameters are used to solve a set of equations, and the solution is then used to derive an expression for the surface impedance Z_S due to the layer's properties and the impedance of the next layer within the sequence. The particle velocity will be expressed from the amplitudes A and B using the relation for plane waves $p = Z_M v$ [K-41].

$$p(x) = A\exp(-ik_C x) + B\exp(ik_C x)$$
$$v(x) = \frac{A}{Z_C}\exp(-ik_C x) - \frac{B}{Z_C}\exp(ik_C x)$$

The material is assumed located with the surface at $x = 0$ and the surface impedance at the front side is given by the ratio of the sound pressure to the particle velocity. The denominator will not equal zero since B is derived from A through reflection (total reflection cannot occur within a physical realisable system due to losses within the boundary layer so the difference is non-zero).

$$Z_S = \frac{p(0)}{v(0)} = \frac{A+B}{A-B} Z_C$$

The impedance at the rear side $x = d$ is derived correspondingly.

$$Z_R = \frac{p(d)}{v(d)} = \frac{A\exp(-ik_C d) + B\exp(ik_C d)}{A\exp(-ik_C d) - B\exp(ik_C d)} Z_C$$

Using the definition of the complex exponential:

$$Z_R = \frac{A\cos(k_C d) - iA\sin(k_C d) + B\cos(k_C d) + iB\sin(k_C d)}{A\cos(k_C d) - iA\sin(k_C d) - B\cos(k_C d) - iB\sin(k_C d)} Z_C$$

Assembling terms:

$$Z_R = \frac{(A+B)\cos(k_C d) - i(A-B)\sin(k_C d)}{(A-B)\cos(k_C d) - i(A+B)\sin(k_C d)} Z_C$$

Dividing with $\cos(k_M d)$ changes the sine into tangent and division with $A - B$ introduces the surface impedance (through Z_S/Z_C from the definition of the surface impedance at $x = 0$).

$$Z_R = \frac{\frac{A+B}{A-B} - i\tan(k_C d)}{1 - i\frac{A+B}{A-B}\tan(k_C d)} Z_C = \frac{\frac{Z_S}{Z_C} - i\tan(k_C d)}{1 - i\frac{Z_S}{Z_C}\tan(k_C d)} Z_C = \frac{Z_S - iZ_C\tan(k_C d)}{Z_C - iZ_S\tan(k_C d)} Z_C$$

Solving for the surface impedance:

$$Z_R Z_C - iZ_R Z_S\tan(k_C d) = Z_S Z_C - iZ_C^2\tan(k_C d)$$
$$Z_C\left(Z_R + iZ_C\tan(k_C d)\right) = Z_S\left(Z_C + iZ_R\tan(k_C d)\right)$$

The surface impedance becomes the rear-side impedance Z_R multiplied by a factor related to the characteristic impedance of the material Z_C, the angular wave number k_C, and the distance d.

$$\boxed{Z_S = \frac{Z_R + iZ_C\tan(k_C d)}{Z_C + iZ_R\tan(k_C d)} Z_C}$$

For iteration: $Z_S = Z_{i+1}$ and $Z_R = Z_i$ where $i = 0$ at the terminating surface (i.e. the rigid wall).

1.3.1. Air cushion

For a rigid wall as the rear-side impedance the surface impedance at distance d from the wall is proportional to a cotangent function. The surface impedance approach infinity for $k_C d = n\pi$, which is at $f_n = nc/2d$, where n = 0, 1, 2, ... Within air this is at f_1 = 1.7 kHz for d = 0.1 m and n = 1.

$$Z_S = \frac{1 + i\frac{Z_C}{Z_R}\tan(k_C d)}{\frac{Z_C}{Z_R} + i\tan(k_C d)} Z_C \xrightarrow[Z_R \to \infty]{} \frac{1}{i\tan(k_C d)} Z_C \quad \Rightarrow \quad \boxed{Z_S = -iZ_C\cot(k_C d)}$$

This equation will be referenced a number of times in the applications chapter.

1.3.2. Compliance

With air as the medium we have $Z_C = \rho_0 c$ and $k_C = k$. At low frequencies the cotangent may be substituted by the first term of the Taylor series expansion: $\cot(x) \approx 1/x - x/3 \ldots$ [RW-198].

$$Z_S = -i\rho_0 c\cot(kd) \xrightarrow[kd \to 0]{} -i\rho_0 c\frac{1}{kd} = \frac{\rho_0 c^2}{i\omega d} = \frac{1}{i\omega C_S} \quad \text{where} \quad C_S = \frac{d}{\rho_0 c^2}$$

The unit of C_S is $m^2\ s^2\ kg^{-1} = m^3/N$, which is the product of compliance (m/N) and area (m^2), and is termed *specific acoustic compliance* [B-35]. The reciprocal is stiffness per unit area. The useful range of this equation is where the second term is insignificant compared to the first term; and this limit will be arbitrarily be set to 3 %.

$$\frac{0.03}{kd} > \frac{kd}{3} \quad \Rightarrow \quad kd < 0.03 \quad \Rightarrow \quad f < \frac{0.3}{2\pi}\frac{c}{d} \quad \Rightarrow \quad f < 0.05\frac{c}{d}$$

The limit corresponds to a layer thickness of less than 5 % of the wavelength or below 160 Hz for a thickness of d = 0.1 m. In other words, the wavelength should be at least twenty times the thickness of the layer for using the compliance approximation.

1.4. Parameters

The flow resistivity is required as input to the Delany and Bazel equations (see section 1.5.1) for estimation of the characteristic impedance and complex wave number of the absorber. The specific mass is needed for determining the acoustical behaviour of the membrane absorber.

1.4.1. Flow resistance

The resistance to air flow through the material is required for determining the effect of fabric placed within the path of sound. The flow resistance is defined as the pressure difference across the sample divided by the particle velocity through it. The value is commonly reported as *flow resistivity* σ, which is the flow resistance per unit length or unit thickness of the sample [CA-169]. The flow resistance R_F is the flow resistivity multiplied by the length.

$$R_F = \frac{\Delta p}{v} \quad \Rightarrow \quad R_F = \sigma d \quad \text{where} \quad \sigma = \frac{\Delta p}{vd}$$

The unit is pressure divided by speed: Pa s/m = Nm^{-3}s = kg m^{-2} s^{-1}, which is also called “rayl” in honour of Lord Rayleigh (1842 – 1919). The unit of flow resistivity is thus Pa s/m^{-2} or rayl m^{-1}. The flow resistance is linear for most materials at sound pressures below 140 dB so non-linearity will not be considered in this report. Some examples of flow resistivity are shown below [CA-170, CA-176] with mineral wool and polyurethane form from Mirowska[4]. The flow resistance for 2 mm of fibrous materials is 400 Pa s/m (σ = 200 kPa s/m^2), which is close to the characteristic impedance of air. Dry sand may be an attractive substitution for use within harsh environments.

Material		**Flow resistivity** $\times 10^3$ Pa s/m^2
Foam	Skum	2…40
Fibrous materials	Fibermaterialer	2…200
Mineral wool, 60 kg/m^3	Mineraluld, 60 kg/m^3	15
Mineral wool, 80 kg/m^3	Mineraluld, 80 kg/m^3	28
Polyurethane foam, 50 mm	Polyuretanskum, 50 mm	50
Forest floor	Skovbund	7…200
Grassland	Græsarealer	70…850
Coarse sand, dry	Groft sand, tørt	50
Fine sand, dry	Fint sand, tørt	150
Loamy sand	Lerblandet sandjord	420
Linestone	Kalksten	800…2500

1.4.2. Specific mass

The specific mass of selected materials[5] at 1mm thickness. Household aluminium foil is 0.01 mm thick so the specific mass becomes 0.027 kg/m^2, and the value is 10 kg/m^2 for glass at 4 mm.

Material	**Mass** (kg/m^3)	**Thickness**	**Specific mass** (kg/m^2)
Gypsum, solid	2787	1 mm	2.8
Aluminium	2700	1 mm	2.7
Glass, window	2579	1 mm	2.6
Plastics	850…1400	1 mm	0.85…1.4
Oak	590…930	1 mm	0.59…0.93
Styroform	30…120	1 mm	0.03…0.12

[4] See: http://www.sea-acustica.es/WEB_ICA_07/fchrs/papers/rba-09-013.pdf

[5] See: http://www.simetric.co.uk/ and http://en.wikipedia.org/wiki/Specific_mass.

1.5. Wave propagation with losses

Sound travelling within air is attenuated but the effect can often be ignored for the calculation of the reverberation time of small rooms [K-160]. However, sound travelling within a medium may be significantly attenuated and this is modelled using a complex value for the angular wave number k. The real part of the wave number defines the phase of the oscillation at the current position as usual, and the imaginary part defines the degree of attenuation. This is shown below where the complex wave number is inserted into the equation of the travelling plane. The product of $-i$ and ik_{Im} is k_{Im} so attenuation with distance requires negative value of k_{Im}.

$$\left.\begin{aligned} p &= p_i \exp(-ikx) \\ k &= k_{\mathrm{Re}} + ik_{\mathrm{Im}} \end{aligned}\right\} \quad \Rightarrow \quad p = p_i \exp(-ik_{\mathrm{Re}}x)\exp(k_{\mathrm{Im}}x) \quad \text{where} \quad \begin{cases} k_{\mathrm{Re}} = \omega/c \\ k_{\mathrm{Im}} < 0 \end{cases}$$

The mechanism behind absorption within fibrous absorbent materials, such as glass and rock wool, is not easily determined so the procedure is usually based upon empirical data. Delany and Bazley undertook a large number of impedance tube measurements and derived the following empirical relationship for the impedance and wave number [CA-173].

1.5.1. Delany and Bazley

When the medium is air, the characteristic impedance is $\rho_0 c$ and the wave number is $k = \omega/c$. For another medium, Delany and Bazley introduce complex correction functions to the parameters for air medium. The characteristic impedance of the medium Z_C is a complex figure multiplied by the characteristic impedance of air, and the complex wave number k_C is a complex figure multiplied by the wave number within air.

$$Z_C = \rho_0 c\left(1 + 0.0571\,X^{-0.754} - i0.087\,X^{-0.732}\right)$$
$$k_C = \frac{\omega}{c}\left(1 + 0.0978\,X^{-0.700} - i0.189\,X^{-0.595}\right)$$

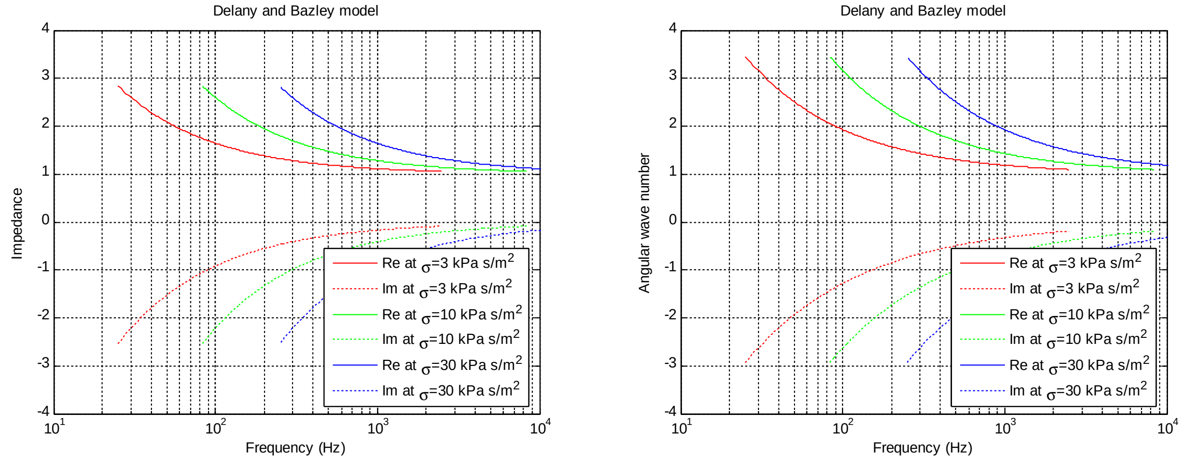
The parameter X is proportional to frequency and is defined by the mass density of air and the flow resistivity of the fibrous absorbent. The parameter is dimensionless and requires that the porosity parameter is close to unity, which is the case with most fibrous absorbers.

$$X = \frac{\rho_0}{\sigma} f \quad \text{where} \quad 0.01 < X < 1$$

The range of X is limited to two decades with an upper limit set through the ratio between flow resistivity and mass density. For a flow resistivity of $\sigma = 20 \cdot 10^3$ Pa s/m^2 the upper limit is 17 kHz thus including a fair part of the audible frequency range, but using the lower limit of $2 \cdot 10^3$ Pa s/m^2 from the table within section 1.4.1 the range becomes upward limited at 1.7 kHz. Using the stated range for X the characteristic impedance and complex wave number becomes:

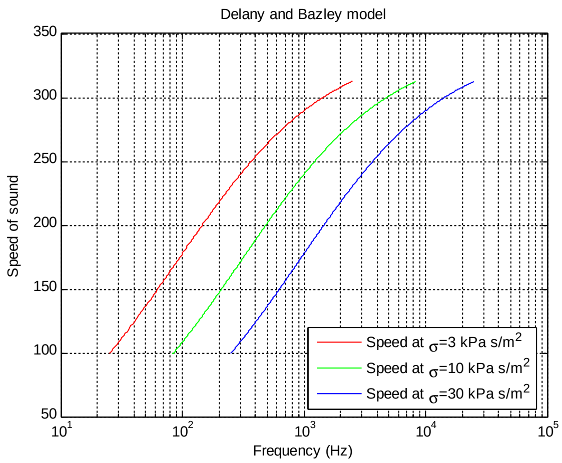
$$Z_C = \rho_0 c(1.06 - i0.09)\ldots\rho_0 c(2.84 - i2.53)$$
$$k_C = \frac{\omega}{c}(1.10 - i0.19)\ldots\frac{\omega}{c}(3.46 - i2.93)$$

The range is from $Z_C \approx 1.06\rho_0 c$ (–5º) and $k_C \approx 1.12\omega/c$ (–10º) at the upper frequency limit so the figures are almost real, while the values are $Z_C \approx 3.8\rho_0 c$ (–42º) and $k_C \approx 4.5\omega/c$ (–40º) at the low-frequency limit with significant negative imaginary parts indicating sound absorption. At the low frequency end there is ample time to level temperature differences between the sound wave and the absorption material.



The characteristic impedance (left) and angular wave number (right) as function of frequency and with the flow resistivity as parameter.

The speed of sound within the material is determined from the real part of the angular wave number as $c = \omega/k_{Re}$. The value is 100 m/s at the low-frequency limit[6] while the higher frequencies approach the speed of sound in air. The low-frequency limit is reached at lower frequency with the lighter mineral wool quality, which is probably due to the reduced interaction between sound particles and the absorbent; hence, more time is required for levelling of temperature differences.



Speed of sound within the absorber as function of frequency and with the flow resistivity as parameter. The real part is shown with full line and the imaginary part is shown dashed.

The Delany and Bazley model used an impedance tube (Kundt's tube) for the measurements, so the incident waves were plane and at normal incidence. In other words, the estimations of characteristic impedance and complex wave number cannot be expected to apply to random incidence without compromising precision. See also the notes within section 1.2.2.

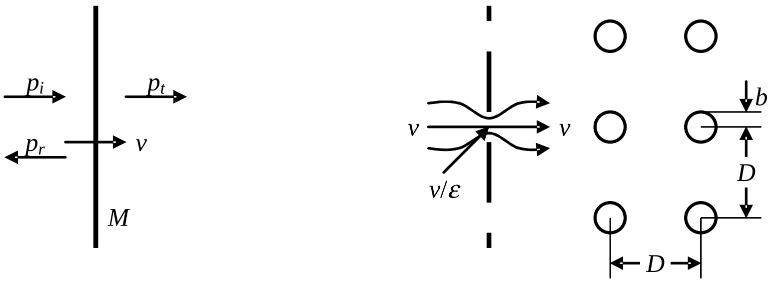
[6] In http://www.international.rockwool-marine.com/applications/sound+insulation/dictionary using option "S" and searching for "Speed of Sound" the value is stated as 180 m/s for Rockwool, which corresponds to the estimated value using 30 kPa s/m^2 and 1 kHz.

2. Applications

Summary – A range of selected examples are presented covering the membrane absorber, which is efficient at low frequencies, and the combinations of fabric and porous absorbers with a rigid wall as support, which are effective at high frequencies. Also analysed is the Helmholtz resonator, which is a combination of porous absorption with resonance absorption at mid frequencies.

2.1. Membrane absorber

A flexible surface is vibrating due to the incident sound and absorbs some of the energy through transmission into the space behind the surface (such as another room). From the acoustical point of view some the sound energy has disappeared, i.e. it has been absorbed. In this respect an open window is an efficient absorber, since it acts as a sink for all the arriving sound energy [K-164].



A membrane absorber is set into oscillation by the pressures acting on the layer thus forcing the air at the rear side to oscillate, which effectively transmits sound energy through the membrane. A perforated membrane performs much the same way using the mass of air within the holes for the oscillation and the higher speed of air corresponds to increased mass.

2.1.1. Thin plate (membrane)

The surface impedance of the membrane shall be determined for a membrane with mass [K-165]. With an incident plane sound wave acting upon the membrane the incident pressure is p_i. Some of the sound is reflected as the sound pressure p_r and the rest is transmitted through the membrane with the sound pressure p_t. The total pressure acting upon the wall is $p_i + p_r - p_t$, and is balanced by the acceleration of the membrane according to Newton's second law $F = M\partial v/\partial t$. The force is pressure multiplied by area $F = pS$ and the time derivative of the speed is $i\omega v$, so $p = i\omega vM/S$. Using m for the mass per unit area ($m = M/S$) the balancing pressure per unit area becomes $i\omega mv$. The particle velocity of the sound at the rear side of the membrane is equal to v, since the air is set into movement by the membrane, so the sound pressure of the transmitted sound is $p_t = \rho_0 cv$. At the left side of the membrane the pressure becomes $p = p_1 + p_2$ so the surface impedance can be estimated.

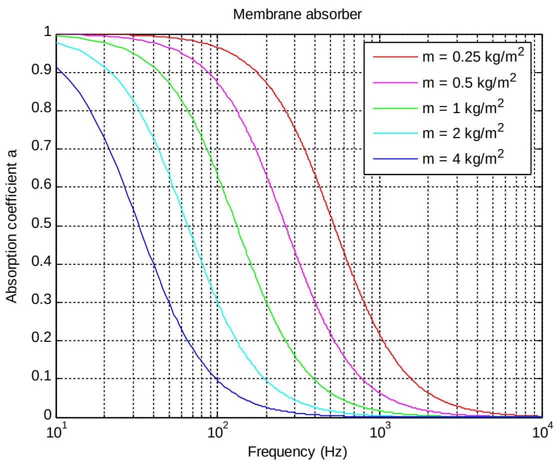
$$p_i + p_r - \rho_0 cv = i\omega mv \quad \Rightarrow \quad p = i\omega mv + \rho_0 cv \quad \Rightarrow \quad \boxed{Z_S = \frac{p}{v} = \rho_0 c + i\omega m}$$

The real term is identical to the characteristic impedance of air, and the imaginary term represents the impedance of mass. The reflection coefficient is determined according to section 1.2.1.

$$R = \frac{Z_S - Z_0}{Z_R + Z_0} = \frac{\rho_0 c + i\omega m - \rho_0 c}{\rho_0 c + i\omega m + \rho_0 c} = \frac{i\omega m}{2\rho_0 c + i\omega m}$$

At low frequencies the reflection coefficient approaches zero, which is close to total absorption, while the asymptotic high-frequency value is unity, so the incident sound is reflected and nothing significant is being absorbed.

The response is shown below with the specific mass of the membrane as parameter. Absorption up to 1 kHz requires a specific mass of less than 0.1 kg/m^2 so thin plastic films are applicable but are also very vulnerable (see section 1.4.2). The plate with specific mass of 4 kg/m^3 is robust but absorption is effective only below approximately 50 Hz.



The response is shown for a membrane absorber with the specific mass as parameter. The value of 0.25 kg/m^2 corresponds to 0.1 mm aluminium foil.

A simple expression is derived for the high-frequency range using the definition from section 1.2.

$$\alpha = 1 - \left| \frac{i\omega m}{2\rho_0 c + i\omega m} \right|^2 = 1 - \left| \frac{i\omega m (2\rho_0 c - i\omega m)}{(2\rho_0 c + i\omega m)(2\rho_0 c - i\omega m)} \right|^2 = 1 - \left| \frac{(\omega m)^2 + 2i\omega m \rho_0 c}{(2\rho_0 c)^2 + (\omega m)^2} \right|^2$$

The square of the magnitude is the sum of the square of the real and imaginary parts.

$$\alpha = 1 - \frac{(\omega m)^4 + (2\omega m \rho_0 c)^2}{\left((2\rho_0 c)^2 + (\omega m)^2\right)^2} = \frac{(2\rho_0 c)^4 + (\omega m)^4 + 2(2\rho_0 c)^2(\omega m)^2 - (\omega m)^4 - (2\omega m \rho_0 c)^2}{(2\rho_0 c)^4 + (\omega m)^4 + 2(2\rho_0 c)^2(\omega m)^2}$$

After reduction it is assumed that the impedance of the wall is much larger than the characteristic impedance of air at high frequencies. The requirement $(2\rho_0 c/\omega m)^2 < 0.1$ corresponds to $f > \rho_0 c/m$. The absorption becomes 0.09 at $f = \rho_0 c/m$ with 0.10 from the approximation.

$$\alpha = \frac{(2\rho_0 c)^4 + (2\omega m \rho_0 c)^2}{(2\rho_0 c)^4 + (\omega m)^4 + 2(2\rho_0 c)^2(\omega m)^2} = \frac{(2\rho_0 c/\omega m)^4 + (2\rho_0 c/\omega m)^2}{(2\rho_0 c/\omega m)^4 + 1 + 2(2\rho_0 c/\omega m)^2} \xrightarrow{\omega m \to \infty} \left(\frac{2\rho_0 c}{\omega m} \right)^2$$

The absorption coefficient is given by the following expression with $m = M/S$ as the specific mass of the plate or membrane, i.e. the mass per unit area [K-165].

$$\alpha = \left(\frac{\rho_0 c}{\pi f m} \right)^2 \quad \text{for} \quad f > \frac{\rho_0 c}{m}$$

For a large glass window with a specific mass of 10 kg/m^2 the equation applies above 40 Hz and the absorption coefficient is 0.02 at 100 Hz. Absorption increases with decreasing frequency, so rooms with very large areas of glass may show significant absorption at very low frequencies [K-166]. The specific mass should thus exceed 10 kg/m^2 for the membrane to perform as a rigid wall.

A thin membrane is assumed to absorb insignificant amount of sound energy, so the sound being absorbed from the primary room is thus transmitted into the secondary room. The transmission level is given by the absorption coefficient, as can be seen from section 1.2 by substituting the absorbed intensity I_a with the transmitted intensity I_t. The transmitted level is $20 \cdot \log_{10}(\alpha)$ dB relative to the incident level. For the above window the level is –35 dB at 100 Hz and –75 dB at 1 kHz compared to the level obtained without the membrane (i.e. by opening the window we have 0 dB loss).

Plastic films may be used as cover to protect the vulnerable part of an absorber, and the signal to be absorbed is thus transmitted through the membrane. For a specific mass of 0.25 kg/m^2, the effect is to gradually reduce the obtainable absorption from the mineral wool above approximately 200 Hz, where the mass of the membrane increases the surface impedance and thus the reflection.

2.1.2. Perforated plate

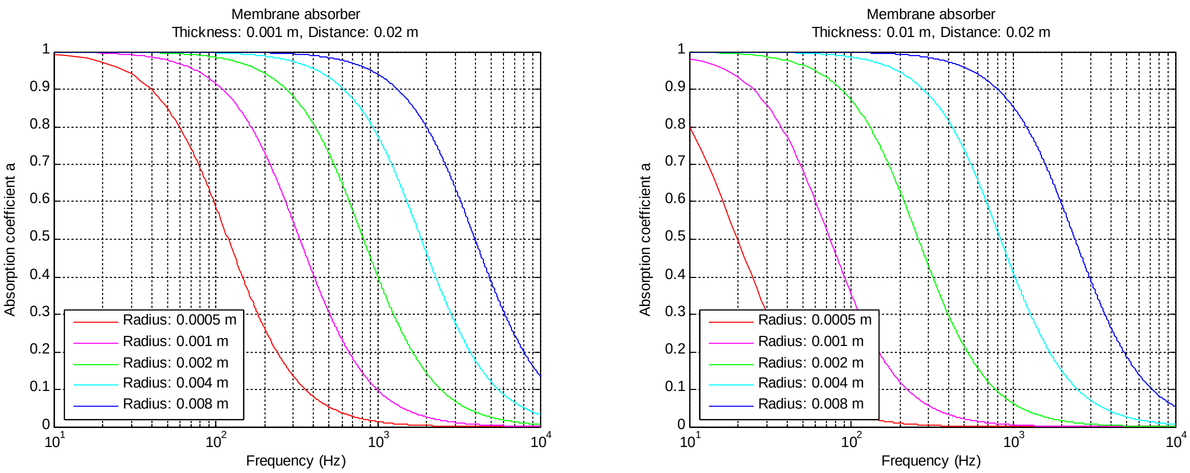
A perforated rigid plate is more robust than a thin membrane and performs in much the same way using the air within the holes as mass. The plate thickness t must be corrected into the effective length L due to the additional mass of air at both ends, which is vibrating in synchronism to the air within the tube. This *end correction* is approximately 0.8b, where b is radius of the hole [K-167].

$$L = t + 1.6b$$

The mass within the tunnel is given by the mass density ρ_0 multiplied by the volume of air, which is contained within a tube with length L and area πb^2; hence, the mass per unit area is $\rho_0 L$. Because of the contraction of the air stream passing through the hole, the air vibrates with a greater velocity than that in the sound wave at some distance from the perforated membrane, so the forces of the air included in the hole are increased by the ratio $D^2/\pi b^2$ where D is the centre to centre distance between the holes, and the effective value of the specific mass increases correspondingly [K-166].

$$m = \frac{D^2}{\pi b^2} \rho_0 L$$

The absorption coefficient is obtained using the equation within section 2.1.1 and the result is shown below with fixed distance between the holes and two plate thicknesses.



The response is shown for a perforated and rigid plate of 5 mm thickness, with the diameter of the hole as parameter and fixed distance of 20 mm between centres of the holes.

The derivation assumes that the wavelength is much larger than the size of the setup since the sound pressure must build-up in order to force the air through the holes. This is comparable to the analysis of compliance from section 1.3.2 where the wavelength should be at least twenty times larger than the largest dimension. The limit is 860 Hz at D = 20 mm, which is not a problem for low-frequency absorption but the analysis should not be pushed far above the limit.

2.2. Porous absorber

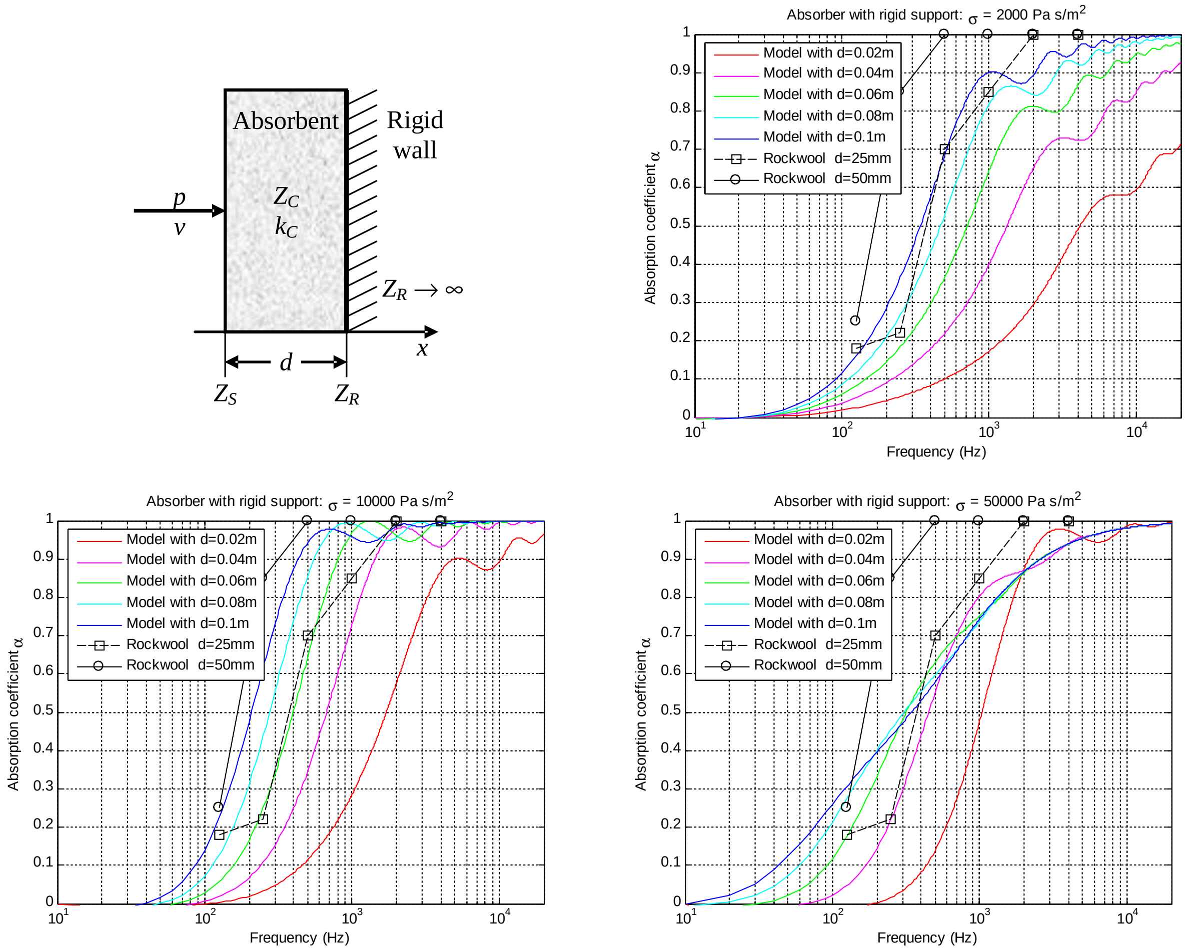
Curtains, carpets and mineral wool are often located near a rigid surface to support the material so the arrangement is an important type of absorber.

2.2.1. Absorbent with rigid support

An absorbent with thickness d is placed against a rigid surface so the surface impedance of the absorbent is estimated as shown within section 1.3.1 for the air cushion and using the characteristic impedance Z_C and complex wave number k_C of the material.

$$Z_S = -iZ_C \cot(k_C d)$$

The characteristic impedance and complex angular wave number are determined from the Delany and Bazley model (see section 1.5). The result is shown below for three values of flow resistivity and compared to data for Rockwool measured at random incidence [CA-157]. The best result is obtained for σ = 10 kPa s/m^2 and the differences may be due to random versus normal incidence.



A porous layer of absorbent with thickness *d* is placed in front of a rigid wall and the absorption coefficient is shown with the material thickness and flow resistivity as parameters. Also shown are random incidence measurements using Rockwool.

A rule-of-thumb states that the material needs to be at least a tenth of a wavelength thick to cause significant absorption, and a quarter of a wavelength to absorb all the incident sound [CA-157]. Using 100 mm of layer and the speed of sound within air, the absorber cut-in frequency is 340 Hz and 860 Hz at full absorption. This is approximately verified for $\sigma = 10$ kPa s/m^2.

2.2.2. Absorbent with air gab

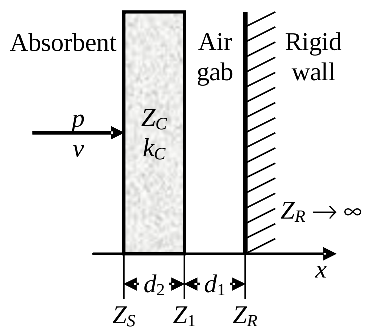
An absorbent with thickness d_2 is placed at distance d_1 from a rigid support. The surface impedance of the air cushion in front of the wall is given by section 1.3.1, and the characteristic impedance of air is $Z_C = \rho_0 c$ so the equation for the air gab becomes:

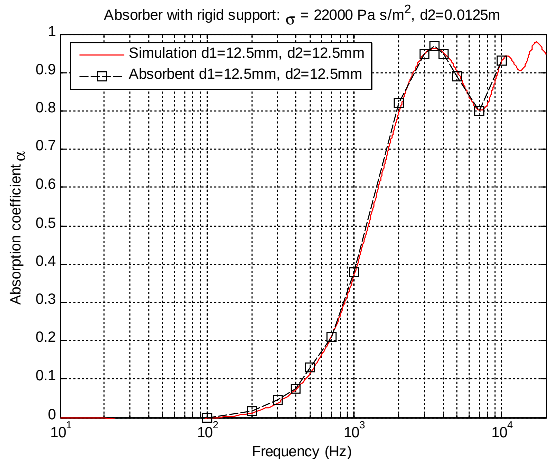
$$Z_1 = -i\rho_0 c \cot(kd_1)$$

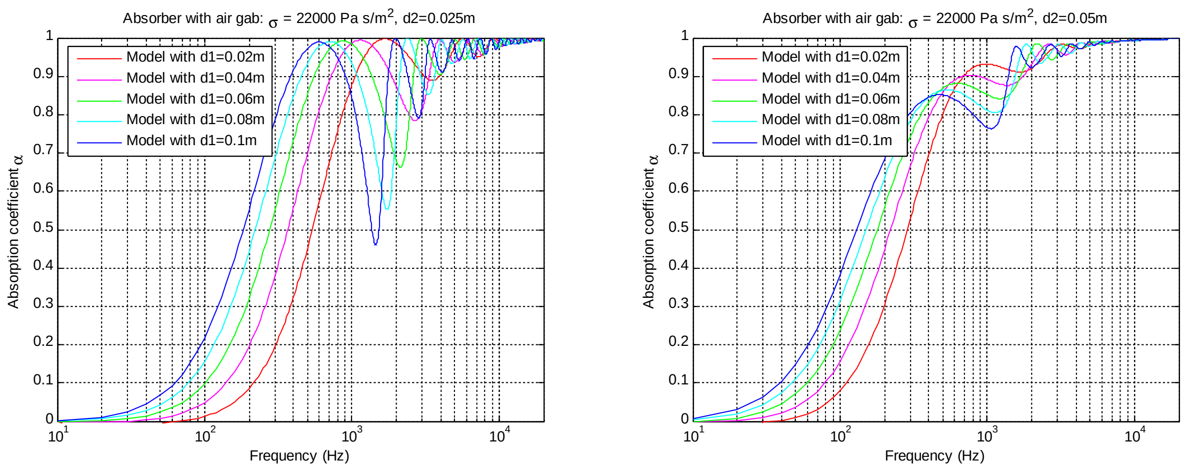
The surface impedance is determined through use of the iterative method (section 1.3) with Z_C and k_C according to the Delany and Bazley model and Z_R substituted by Z_1 from the above equation.

$$Z_S = \frac{Z_1 + iZ_C \tan(k_C d_2)}{Z_C + iZ_1 \tan(k_C d_2)} Z_C$$

The simulation below was compared to reported figures using similar theory to check the algorithm and also becoming acquainted to the nomenclature using porous absorbers [CA-185]. The best fit to data was obtained at $\sigma = 22 \cdot 10^3$ Pa s/m^2 so this value was selected for the analysis.







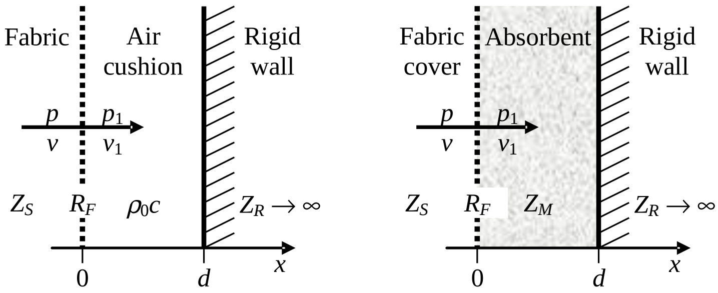
Top picture is absorbent thickness 12.5 mm on 12.5 mm air gab. Bottom pictures are with fixed absorbent thickness d_2 of 25 mm or 50 mm and placed at distance d_1 from a rigid support.

The air cushion generates zero impedance at half wavelength, 7 kHz for d_1 = 12.5 mm (top figure), and 1.7 kHz for d_1 = 0.1 m (blue line, bottom left). For d_1 = 0.02 m the notch should ideally move to 8.6 kHz but the red curve shows a notch at approximately 3.5 kHz, so the boundary between the mineral wool and the air cushion is not as sharply defined as expected.

2.3. Fabric

Often an absorbent is finished with a thin porous layer to make the absorber look better or improve its robustness, but the application also includes curtains, lowered ceiling using thin fibrous materials for sound absorption and canvas on frame such as images.

In this section three setups will be analysed. At first a porous fabric is allowed to let the air move but the layer itself remains at rest, the second allows the fabric to vibrate, and the third includes an absorbing material behind the fabric.



A porous layer of thin fabric located at distance *d* from a rigid wall (left), and also with absorbent filling between the fabric and wall (right). The porous fabric has the flow resistance R_F and the impedance of the space between the fabric and wall is either the characteristic impedance of air or the impedance of the absorbent material.

2.3.1. Fabric with air cushion

The absorption coefficient is determined for a thin porous layer of fabric is hung at distance d from a rigid wall and it is assumed that the fabric is so heavy that it does not vibrate under the influence of the incident sound. Any pressure difference between the two sides of the fabric forces an air stream through the pores of the fabric with air velocity v_S, and it is assumed that the relation is valid for a steady flow of air as well as for an alternating flow. The flow resistance of the porous layer is defined in an analogue manner to Ohm's law within electrical circuits, i.e. the pressure difference across the fabric $p - p_1$ is equal to the particle velocity v_S through the surface multiplied by the flow resistance R_F of the fabric [K-45]. See section 1.4.1 for typical flow resistance values.

$$R_F = \frac{p - p_1}{v_S} \quad \Rightarrow \quad p = R_F v_S + p_1$$

Because of conservation of matter, the particle velocities at both sides of the layer must be equal to each other thus avoiding build-up of air particles. This is analogous to Kirchhoff's node-law within the field of electrical circuitry, which states that the flow into a node must equal the flow out from the node. The particle velocities at all positions near the fabric are thus equal.

$$v_S = v(0) = v_1(0)$$

The surface impedance at $x = 0$ is calculated from the sound pressure and the particle velocity at this location. The ratio p_1/v_1 represents the impedance of the air cushion between the fabric and the rigid wall, which is given by section 1.3.1 using $Z_M = \rho_0 c$ for the characteristic impedance of air.

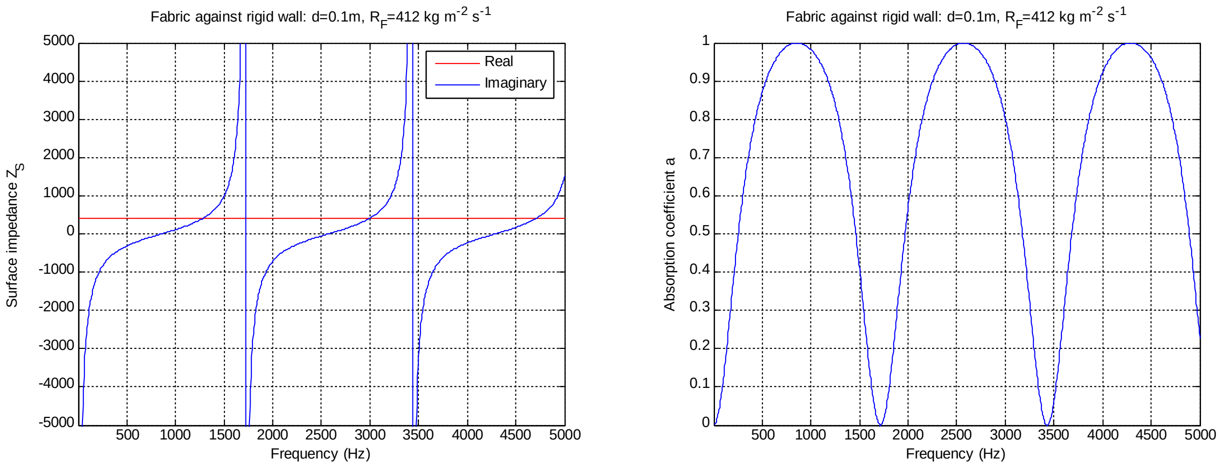
$$Z_S = \frac{p}{v} = R_F + \frac{p_1}{v_1} \quad \Rightarrow \quad \boxed{Z_S = R_F - i\rho_0 c \cot(kd)}$$

The result of the design is shown below assuming a flow resistance of $R_F = \rho_0 c$, which corresponds to 2 mm thickness using tight fibrous materials ($d = R_F/\sigma$ and $\sigma = 200$ kPa s/m^2, see section 1.4.1).

The absorption coefficient oscillates between zero and full absorption at discrete frequencies. There is full absorption at the frequencies where the cotangent is zero, which occur for an argument of $\pi/2$ plus an integer multiple of π, which is at f_0 = 858 Hz for n = 0 and f_1 = 2.57 kHz for n = 1, etc. This is where the distance between fabric and wall is odd multiple of one-quarter of the wavelength since the particle velocity amplitude is at its maximum through the fabric.

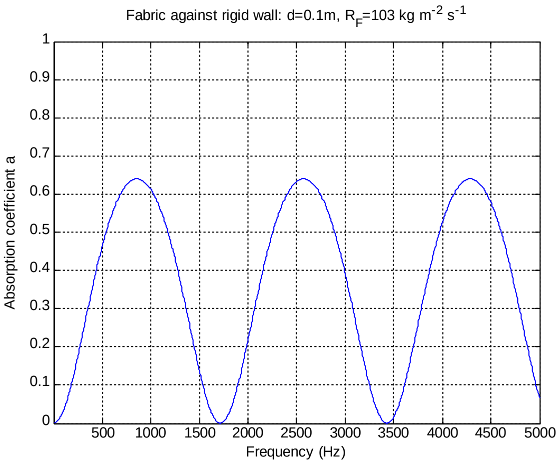
$$kd = \frac{\pi}{2} + n\pi = \frac{1+2n}{2}\pi \quad \Rightarrow \quad f_n = \frac{(1+2n)c}{4d}$$

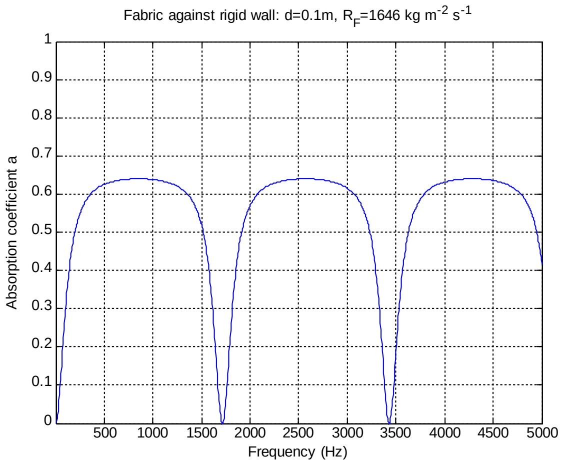
The absorption is zero at the frequency where the cotangent approaches infinity, which is at zero frequency, $2f_0$ = 1.72 kHz, $4f_0$ = 3.43 kHz, etc. This is where the distance between fabric and wall is an integral multiple of half the wavelength since the particle velocity is near zero at the surface of the fabric (and at the wall) so any significant energy loss cannot take place [K-47].



The complex impedance of a fabric placed d = 0.1 m in front of a rigid wall (left) and the absorption coefficient (right) for the flow resistance value $R_F = \rho_0 c$.

The absorption coefficient is reduced for other values of the flow resistance since the impedance does not match the impedance of air. Examples are shown for a flow resistance of one-quarter of the characteristic value of air and also for at four times the characteristic value. The absorption at the peak values is reduced to 0.65, so the design is robust to manufacturing tolerances. For broadband absorption the flow resistance should be designed larger than the characteristic impedance of air. The distance can be modulated by folding the curtain to reduce the ripple through averaging.





Absorption coefficient with the flow resistance value $R_F = 0.25\rho_0 c$ (left) and $R_F = 4\rho_0 c$ (right).

2.3.2. Fabric with mass

A porous layer will not remain at rest in a sound field but vibrates with velocity v_M. According to Newton's second law acceleration of the fabric is given by the force F acting upon it and the mass of the fabric M. The force is the difference between the sound pressures multiplied by the area of the fabric, and acceleration is the time derivative of the particle velocity. The ratio M/S represents the specific mass of the fabric, i.e. mass per unit area, and the assumption of harmonic oscillations simplifies differentiation into $i\omega$. The direction is positive along the x-axis.

$$F = M\frac{\partial v_M}{\partial t} \quad \text{where} \quad F = (p - p_1)S \quad \Rightarrow \quad p - p_1 = \frac{M}{S}\frac{\partial v_M}{\partial t} = i\omega m v_M$$

The particle velocity at the surface of the fabric v_S is increased by the velocity of the fabric v_M.

$$v_S = \frac{p - p_1}{R_F} + v_M \quad \Rightarrow \quad v_M = v_S - \frac{p - p_1}{R_F} \quad \Rightarrow \quad p - p_1 = i\omega m\left(v_S - \frac{p - p_1}{R_F}\right)$$

The flow impedance Z_F is the pressure difference across the fabric divided by the particle velocity through the fabric. The ratio R_F/m is characteristic for the fabric and defines the transition from low frequencies where the fabric oscillates to high frequencies where the fabric is at rest.

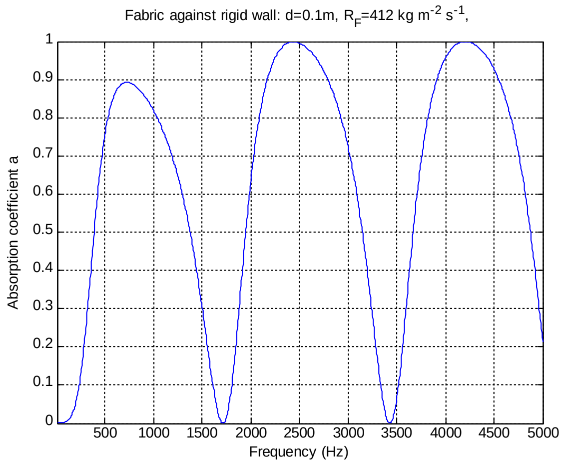
$$(p - p_1)\left(1 + \frac{i\omega m}{R_F}\right) = i\omega m v_S \quad \Rightarrow \quad Z_F = \frac{p - p_1}{v_S} = \frac{i\omega m}{1 + \frac{i\omega m}{R_F}} = \frac{i\omega}{i\omega + \omega_S} R_F \quad \text{where} \quad \omega_S = \frac{R_F}{m}$$

The cross-over frequency is $f_S = \omega_S/2\pi = 655$ Hz using the values $R_F = \rho_0 c$ and $m = 0.1$ kg m^{-2}.

The effect of the flow impedance can thus be ignored at low frequencies. The surface impedance is found by substituting R_S with Z_S using the equation from the previous result [K-48].

$$\boxed{Z_S = \frac{i\omega}{i\omega + \omega_S} R_F - i\rho_0 c \cot(kd)}$$

This equation is plotted below using $R_F = \rho_0 c$ and $m = 0.1$ kg m^{-2} and shows a reduction of the sound absorption at low frequencies where the fabric is vibrating with the sound wave thus reducing the particle velocity through the material.



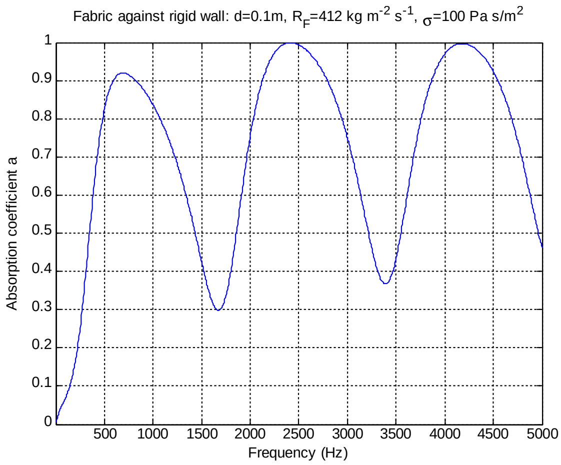
The effect of including acceleration of the mass of the fabric into the equation is to reduce the absorption coefficient at low frequencies.

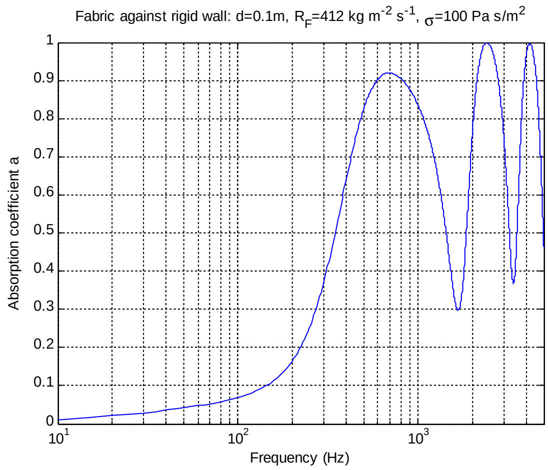
2.3.3. Fabric with absorbent

The air gab is now assumed filled with an absorbent material thus changing the impedance of the air gab from the characteristic impedance of air $\rho_0 c$ into the characteristic impedance of the material Z_C with the complex wave number k_C according to Delany and Bazley (see section 1.5.1). It is further assumed that the fabric is not loaded by the porous absorbent but is allowed to oscillate.

$$Z_S = \frac{i\omega}{i\omega + \omega_S} R_F - iZ_C \cot(k_C d)$$

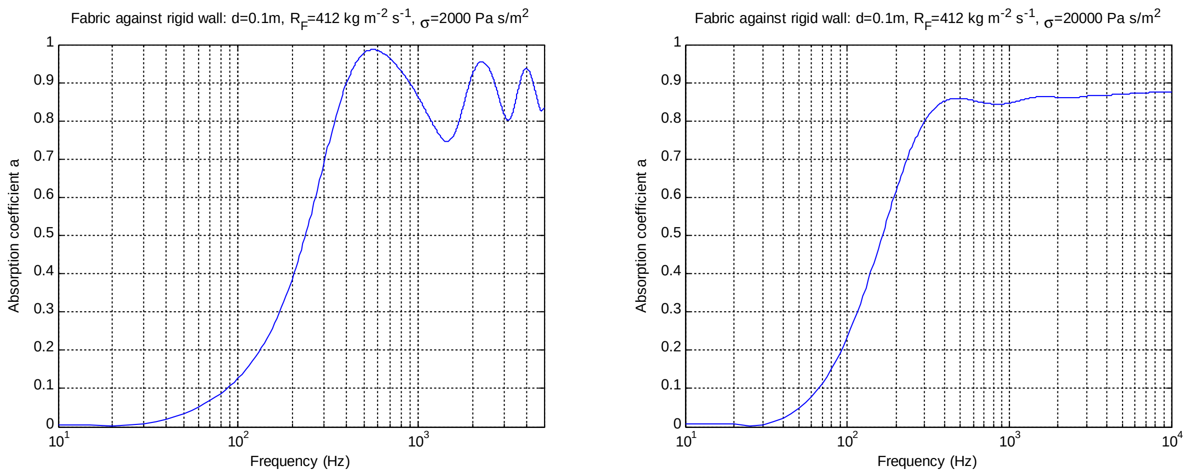
The result is shown below with $R_F = \rho_0 c$ and σ =100 Pa s/m^2, which represents almost no absorbent material being present and the effect is to reduce the anti-resonance dips although the response is virtually unaffected. High-frequency absorption is reduced with $R_F > \rho_0 c$ and the peak frequency lowered, while the high frequencies absorption approaches unity for $R_F < \rho_0 c$ while the main peak approaches 1 kHz. Two plots are shown with different resolution for the frequency axes.





Absorption coefficient versus frequency for fabric located at 100 mm distance from a rigid support with an absorbent material with almost no absorption. The two plots report the same response.

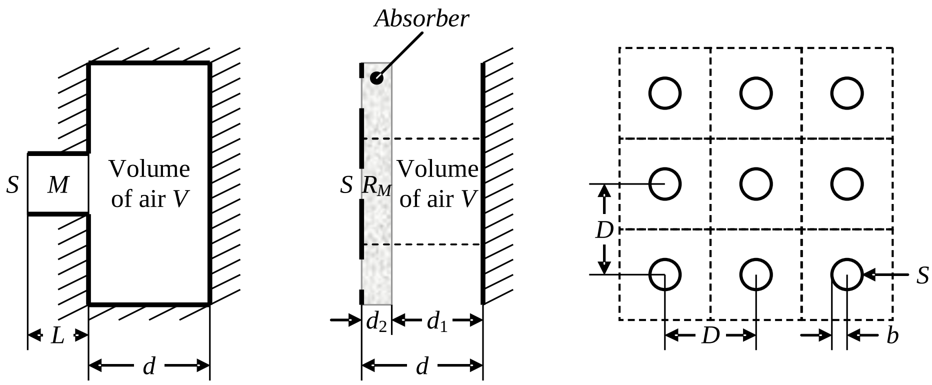
The next plot uses the flow resistivity of 2 kPa s/m^2 for the left plot showing reduced oscillation and 20 kPa s/m^2 for the right plot where oscillations are virtually eliminated. The absorption coefficient is around 0.85 from 300 Hz and up but comparison to real measurements would be nice.



Absorption coefficient versus frequency for fabric located at 100 mm distance from a rigid support with an absorbent material with absorption corresponding to mineral wool.

2.4. Helmholtz resonator

The conventional Helmholtz resonator is a mass and compliance system with a characteristic frequency of oscillation, which is first determined from a mechanical analogy. The model is then modified into the common application for acoustical use with a perforated plate. An absorber is placed close to the perforated sheet where the particle velocity is large.



A Helmholtz resonator consists of a mass and a compliance system with damping from an absorber. The resonator is commonly implemented with a perforated plate and a sheet of absorbent material.

2.4.1. Resonance frequency

The Helmholtz resonator consists of the mass of air within the tube and the compliance due to the volume of air within the container (see section 1.3.2). The mass of air within the tube is the mass density of air multiplied by the volume of the tube given by the tube length L and the cross sectional area S of the tube with b as radius[7].

$$M_M = \rho_0 LS = \rho_0 L\pi b^2$$

If the mass moves distance x into the resonator the air pressure increases ∂P above the nominal level P_0 due to the decrease of volume, which is given by the product of cross-sectional area of the piston and the distance x as: $\partial V = \partial Sx$. The change in pressure is not reciprocal to the change in volume since the process is adiabatic; this means that the change is assumed so fast that the increase

[7] From http://www.phys.unsw.edu.au/jw/Helmholtz.html.

within temperature cannot level out and the temperature oscillates as well. The relation between change within pressure and volume is defined by the law according to Biot and Savart. From this the relation between the increase in pressure ∂P for the air moving distance x is found.

$$\frac{\partial P}{P_0} = -\gamma \frac{\partial V}{V} = -\gamma \frac{Sx}{V} \quad \Rightarrow \quad \partial P = -\gamma \frac{P_0 S}{V} x$$

According to Newton's second law the mass is accelerated by the force acting upon the mass. This force is the contained air, which counteracts the increase of pressure. The force acting upon the air within the tube is the change in pressure ∂P multiplied by the cross-sectional area S of the tube.

$$F = M_M \frac{\partial^2 x}{\partial t^2} \quad \Rightarrow \quad \frac{\partial^2 x}{\partial t^2} = \frac{F}{M_M} = \frac{\partial PS}{\rho_0 SL}$$

Insertion of ∂P into the equation and assuming harmonic oscillation (so differentiation corresponds to multiplication with $i\omega$) produces an equation for determination of the characteristic frequency of oscillation, given by the ratio of specific heat ($\gamma = 1.4$ for diatomic gasses), the static pressure at the surface of earth ($P_0 = 100$ kPa), the cross-sectional area of the tube $S = \pi b^2$, the mass density of air at 20ºC $\rho_0 = 1.2$ kg/m^3, the volume of contained air V and the length of the tube L. The ratio $\gamma P_0/\rho_0$ is the speed of sound in air squared [B-22].

$$\frac{\partial^2 x}{\partial t^2} = -\gamma \frac{P_0 S}{\rho_0 VL} x \quad \Rightarrow \quad (i\omega)^2 x = -\gamma \frac{P_0 S}{\rho_0 VL} x \quad \Rightarrow \quad \omega^2 = \gamma \frac{P_0 S}{\rho_0 VL} \quad \Rightarrow \quad \boxed{f = \frac{c}{2\pi}\sqrt{\frac{S}{VL}}}$$

For a bottle with $S = 1$ cm^2, V = 0.33 dm^3 and $L = 5$ cm the resonance frequency becomes $f = 134$ Hz.

2.4.2. Perforated plate

The length parameter should include the *end correction*, which represents the mass of air oscillating in synchronism with the air within the tube, and this was stated as 0.8 times radius of the tube for each end within section 2.1.2. This correction applies to one single hole within a large surface and taking mutual interactions into account the end correction factor δ should rather be calculated as shown below for each end of the tube. For a perforated sheet the hole area is $S = \pi b^2$ and the volume for each hole is $V = D^2 d$. In the expression below the ratio S/D^2 has been substituted by the porosity ε, which is the fraction of open area of the perforated sheet [CA-210].

$$\boxed{f = \frac{c}{2\pi}\sqrt{\frac{\varepsilon}{d(t+2\delta b)}} \quad \text{where} \quad \varepsilon = \frac{\pi b^2}{D^2} \quad \text{and} \quad \delta = 0.8\left(1 - 1.4\sqrt{\varepsilon}\right)}$$

For a perforated sheet with radius of the hole $b = 1$ mm and $D = 10$ mm between the centre of the holes, the porosity is $\varepsilon = 0.031$, so the end correction becomes $\delta = 0.60$. For sheet thickness $t = 10$ mm and depth from sheet to the rigid support of $d = 0.1$ m, the resonance frequency becomes $f = 290$ Hz.

2.4.3. Surface impedance

The surface impedance is calculated in three steps. The impedance of the air cushion is given by section 1.3.1 using the characteristic impedance of air $Z_C = \rho_0 c$ and the thickness d_1 of the air cushion and is labelled Z_1. The impedance of the absorption layer is determined using the iteration equation from section 1.3 with Z_1 as the rear side impedance and Z_C and k_C for the absorption material using the Delany and Bazley model, and the surface impedance at this point is labelled Z_2.

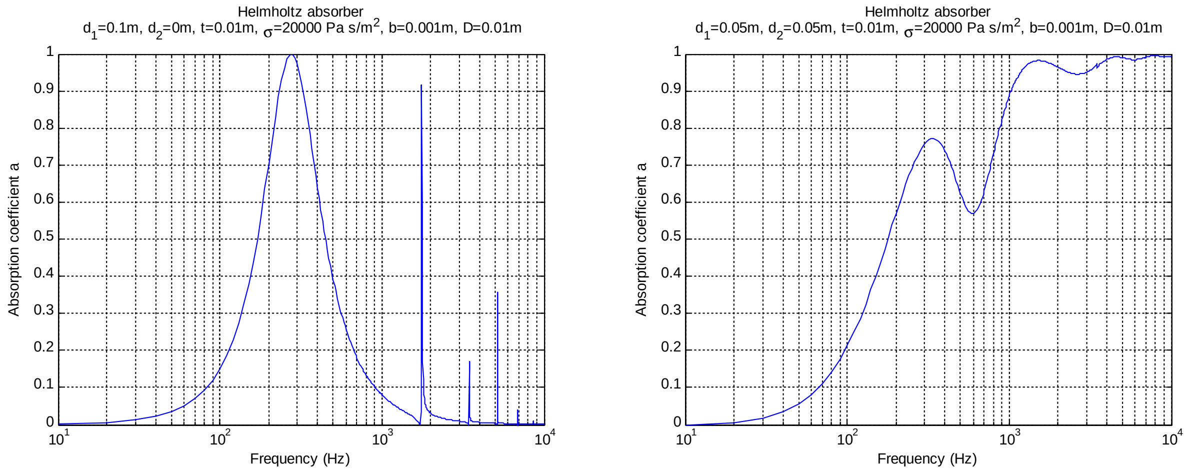
Finally the surface impedance at the perforated plate is calculated by addition of the impedance of the mass of air *m* from the perforated plate.

$$Z_S = i\omega m + Z_2, \qquad Z_2 = \frac{Z_1 + iZ_C \tan(k_C d_2)}{Z_C + iZ_1 \tan(k_C d_2)} Z_C, \qquad Z_1 = -i\rho_0 c \cot(k d_1)$$

The mass from the holes within the perforated sheet is determined as the effective mass per unit area for a plate with thickness *t*, hole radius *b* and distance *D* between the centre of the holes.

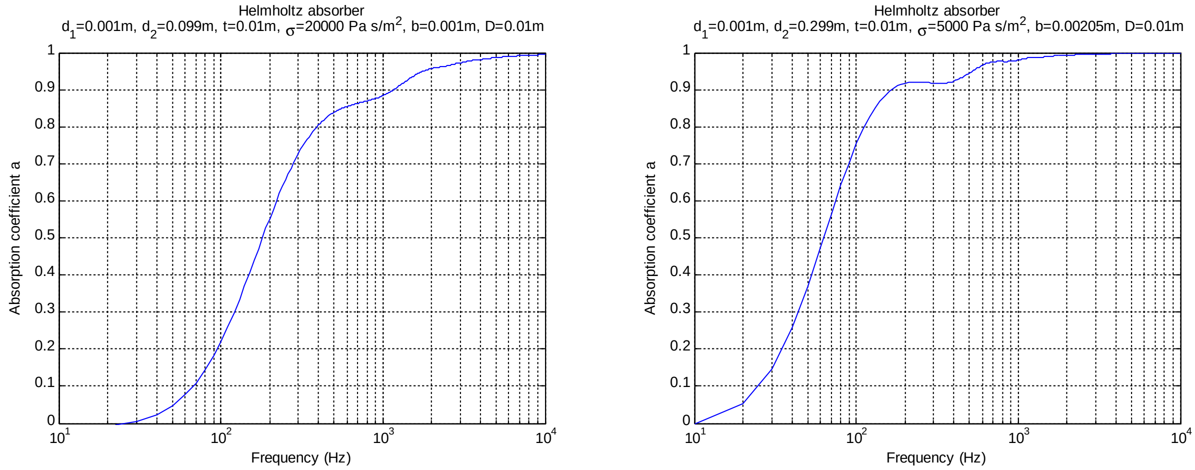
$$m = \frac{D^2}{\pi b^2} \rho_0 (t + 2\delta b)$$

Using the example values from the previous section, the resonance frequency should equal 290 Hz using 10 mm plate thickness, 1 mm radius of the hole, 10 mm spacing centre to centre between the holes and 100 mm from the rear of the perforated plate to the rigid surface. The result with this setup is shown to the left below. The absorption is close to unity within a narrow frequency band around the resonance frequency. The introduction of absorption material is shown to the right with the volume half filled with mineral wool located close to the perforated plate. The absorption coefficient is 0.75 at the resonance frequency, which is increased in frequency to 350 Hz, but the absorption coefficient is above 0.55 for frequencies above 200 Hz and close to unity above 1 kHz.



To the left is shown the absorption from the Helmholtz resonator without absorption and ignoring the viscous losses due to the air within the tube. To the right is shown the effect of introducing some absorption material to the front half of the volume.

Increasing the thickness of the absorbent the result becomes comparable to an absorbent without the perforated plate (see section 2.2.1). Increasing the depth of the arrangement and using lighter absorption material improves low frequency absorption but the construction becomes thicker and occupies more of the valuable space.



To the left is the volume completely filled with absorption material and to the right is the depth of the construction increased and the absorbing material is lighter.

2.5. Conclusion

A membrane absorbs low frequencies by transmitting some of the energy into a neighbouring room while high frequencies are reflected due to the mass. The perforated sheet performs much in the same manner although using the air within the holes as mass. An absorbent placed close to a rigid wall results in high frequencies being absorbed while the low frequencies are reflected. With air gab before the wall a series of notches at integral multiples of half wave lengths are created. This is also the effect with the thin porous fabric hung in front of a rigid wall and the oscillations are reduced by folding the fabric. The Helmholtz resonator introduces flexibility in modifying the low frequency response but the major benefit appears to be the introduction of a protective plate.

3. MATLAB code

3.1. Delany and Bazel model

```
% DelanyAndBazley.m
clear
c=343;                          % Speed of sound (m/s).
rho=1.2;                        % Density of air (kg m-3).
sigma=[3000 10000 30000];       % Flow resistivity (Pa s/m2).
C=['r' 'g' 'b'];
for e=1:3
    fmax=sigma(e)/rho;          % Max frequency of the model (Hz).
    fmin=fmax/100;              % Min frequency of the model (Hz).
    f=fmin:fmax;                % Frequency axis (Hz).
    X=f/fmax;                   % Quantity for Delany and Bazley.
    disp(['Max frequency: ' num2str(fmax) ' at sigma: ' num2str(sigma(e))])
    ZC=1+0.0571*(X.^-0.754)-i*0.087*(X.^-0.732);    % Normalised impedance.
    kC=1+0.0978*(X.^-0.700)-i*0.189*(X.^-0.595);    % Normalised wavenumber.
    semilogx(f,abs(c./real(kC)),'Color',C(e),'LineStyle','-') % Plot speed of sound.
    %semilogx(f,real(ZC),'Color',C(e),'LineStyle','-')   % Plot real part.
    hold on
    %semilogx(f,imag(ZC),'Color',C(e),'LineStyle',':')   % Plot imaginary part.
end
hold off
title('Delany and Bazley model')
xlabel('Frequency (Hz)')
ylabel('Speed of sound')
legend(['Speed at \sigma=' num2str(sigma(1)/1000) ' kPa s/m^2'], ...
       ['Speed at \sigma=' num2str(sigma(3)/1000) ' kPa s/m^2'], ...
       ['Speed at \sigma=' num2str(sigma(3)/1000) ' kPa s/m^2'],'Location','SouthEast')
%ylabel('Impedance')
%ylabel('Angular wave number')
%legend(['Re at \sigma=' num2str(sigma(1)/1000) ' kPa s/m^2'], ...
```

```
%       ['Im at \sigma=' num2str(sigma(1)/1000) ' kPa s/m^2'], ...
%       ['Re at \sigma=' num2str(sigma(2)/1000) ' kPa s/m^2'], ...
%       ['Im at \sigma=' num2str(sigma(2)/1000) ' kPa s/m^2'], ...
%       ['Re at \sigma=' num2str(sigma(3)/1000) ' kPa s/m^2'], ...
%       ['Im at \sigma=' num2str(sigma(3)/1000) ' kPa s/m^2'],'Location','SouthEast')
%axis([10 10000 -4 4])
grid on
```

3.2. Membrane absorber

```
% MembraneAbsorber.m
clear
t=1e-3;                         % Sheet thickness m).
D=20e-3;                        % Hole spacing (m).
c=343;                          % Speed of sound within air (m/s1).
rho=1.2;                        % Density of air (kg/m3).
f=10:2:10000;                   % Frequency axis (Hz).
k0=2*pi*f/c;                    % Angular wave number (m-1).
Z0=rho*c;                       % Impedance of air (kg m-2 s-1).
C=['r','m','g','c','b'];
for n=1:5;                      % Mass per unit area (kg/m2).
    b(n)=(1e-3)*2^(n-2);        % Hole radius (m).
    L=t+1.6*b(n);               % Effective length (m).
    %m(n)=2^(n-1)/4;                 % (1) Rigid plate (kg/m2).
    m(n)=(D^2/(pi*b(n)^2))*rho*L;   % (2) Perforated plate (kg/m2).
    ZS=Z0+i*2*pi*f*m(n);        % Impedance of air cushion (kg m-2 s-1).
    R=(ZS-Z0)./(ZS+Z0);         % Reflection coefficient.
    a=1-abs(R).^2;              % Absorption coefficient.
    semilogx(f,a,'Color',C(n))
    hold on
end
hold off
title({'Membrane absorber',['Thickness: ' num2str(t) ' m, Distance: ' num2str(D) ' m']})
xlabel('Frequency (Hz)')
ylabel('Absorption coefficient a')
%legend(['m = ' num2str(m(1)) ' kg m-2'],['m = ' num2str(m(2)) ' kg m-2'], ...
%       ['m = ' num2str(m(3)) ' kg m-2'],['m = ' num2str(m(4)) ' kg m-2'], ...
%       ['m = ' num2str(m(5)) ' kg m-2'],'Location','SouthWest')
legend(['Radius: ' num2str(b(1)) ' m'],['Radius: ' num2str(b(2)) ' m'], ...
       ['Radius: ' num2str(b(3)) ' m'],['Radius: ' num2str(b(4)) ' m'], ...
       ['Radius: ' num2str(b(5)) ' m'],'Location','SouthWest')
grid on
```

3.3. Absorbent with rigid support

```
% AbsorbentWithRigidSupport.m
clear
c=343;                      % Speed of sound within air (m/s).
rho=1.2;                    % Density of air (kg/m3).
sigma=50000;                % Flow resistivity (Pa s/m2).
disp(['Max frequency: ' num2str(sigma/rho)])
Z0=rho*c;                   % Impedance of air (Pa s/m).
f=10:10:20000;              % Frequency axis (Hz).
X=(rho/sigma)*f;            % Quantity for Delany and Baxley.
ZC=rho*c*(1+0.0571*(X.^-0.754)-i*0.087*(X.^-0.732));
kC=(2*pi*f/c).*(1+0.0978*(X.^-0.700)-i*0.189*(X.^-0.595));
C=['r','m','g','c','b'];
for e=1:5
    d(e)=e*0.02;
    ZS=-i*ZC.*cot(kC*d(e)+eps);     % Flow resistance.
    R=(ZS-Z0)./(ZS+Z0);
    a=1-abs(R).^2;
    semilogx(f,a,'Color',C(e))
    hold on
end
RW25mm=[0.18 0.22 0.7  0.85 1    1];    % Absorption coefficients
RW50mm=[0.25 0.85 1    1    1    1];    % for Rockwool according to
RWF=   [125  250  500  1000 2000 4000]; % Cox & D'Antonio (page 157).
semilogx(RWF,RW25mm,'--sk')
semilogx(RWF,RW50mm,'-ok')
hold off
title(['Absorber with rigid support: \sigma = ' num2str(sigma) ' Pa s/m^2'])
xlabel('Frequency (Hz)')
```

```
ylabel('Absorption coefficient \alpha')
legend(['Model with d=' num2str(d(1)) 'm'], ['Model with d=' num2str(d(2)) 'm'], ...
       ['Model with d=' num2str(d(3)) 'm'], ['Model with d=' num2str(d(4)) 'm'], ...
       ['Model with d=' num2str(d(5)) 'm'], ...
        'Rockwool  d=25mm','Rockwool  d=50mm','Location','NorthWest')
grid on
axis([10 20000 0 1])
```

3.4. Fabric near rear wall

```
% FabricNearRigidWall.m
clear
c=343;                      % Speed of sound within air (m/2).
rho=1.2;                    % Density of air (kg/m3).
sigma=20000;                % Flow resistivity (Pa s/m2).
d=0.10;                     % Thickness of absorbent (m).
MS=0.1;                     % Specific mass of fabric (kg m-2).

f=0:5:5000;                 % Frequency axis (Hz).
k=2*pi*f/c;                 % Angular wave number (m-1).
Z0=rho*c;                   % Impedance of air (kg m-2 s-1).
RF=1.0*Z0;                  % Flow resistance (kg m-2 s-1).
wS=RF/MS;                   % Cut-off frequency (rad/s).

% Flow resistance with fixed curtain.
%ZS=RF-i*rho*c*cot(k*d+eps);

% Flow resistance with vibrating curtain (finite mass).
%ZS=(i*2*pi*f./(i*2*pi*f+wS))*RF-i*rho*c*cot(k*d+eps);

% Flow resistance with vibrating curtain and absorbent.
X=(rho/sigma)*f;            % Quantity for Delany and Baxley.
ZC=rho*c*(1+0.0571*(X.^-0.754)-i*0.087*(X.^-0.732));
kC=(2*pi*f/c).*(1+0.0978*(X.^-0.700)-i*0.189*(X.^-0.595));
ZS=(i*2*pi*f./(i*2*pi*f+wS))*RF-i*ZC.*cot(kC*d+eps);

% Surface impedance.
figure(1)
plot(f,real(ZS),'-r', f,imag(ZS),'-b')
title(['Fabric against rigid wall: d=' num2str(d) 'm, ' ...
       'R_F=' num2str(round(RF)),' kg m^{-2} s^{-1}'])
xlabel('Frequency (Hz)')
ylabel('Surface impedance Z_S')
legend('Real','Imaginary')
grid on
axis([1 5000 -5000 5000])

% Reflecion coefficient.
figure(2)
R=(ZS-Z0)./(ZS+Z0);
plot(f,real(R),'-r', f,imag(R),'-b')
title(['Fabric against rigid wall: d=' num2str(d) 'm, ' ...
       'R_F=' num2str(round(RF)),' kg m^{-2} s^{-1}'])
xlabel('Frequency (Hz)')
ylabel('Reflection coefficient R')
axis([1 5000 -1 1])

% Absorption coefficient.
figure(3)
a=1-abs(R).^2;
plot(f,abs(a))
%semilogx(f,abs(a))
title(['Fabric against rigid wall: d=' num2str(d) 'm, ' ...
       'R_F=' num2str(round(RF)),' kg m^{-2} s^{-1}, ' ...
       '\sigma=' num2str(sigma),' Pa s/m^2'])

xlabel('Frequency (Hz)')
ylabel('Absorption coefficient a')
grid on
axis([10 5000 0 1])
```

3.5. *Helmholtz absorber*

```
% HelmholtzAbsorber.m
clear
t=10e-3;                     % Sheet thickness m).
d1=0.001;                    % Thickness of air cushion (m).
d2=0.099;                    % Thickness of absorbent (m).
b=1.0e-3;                    % Hole radius (m).
D=10e-3;                     % Hole spacing (m).
c=343;                       % Speed of sound within air (m s-1).
rho=1.2;                     % Density of air (kg m-3).
sigma=10000;                 % Flow resistivity (Pa s/m2).
e=pi*b^2/D^2;                % Porosity.
delta=0.8*(1-1.4*sqrt(e));  % End correction.
fres=(c/(2*pi))*sqrt(e/((d1+d2)*(t+2*delta*b)))

f=10:10:10000;               % Frequency axis (Hz).
k0=2*pi*f/c;                 % Angular wave number (m-1).
Z0=rho*c;                    % Impedance of air (kg m-2 s-1).
m=rho*D^2*(t+2*delta*b)/(pi*b^2);    % Mass of air block (kg m-2).
Z1=Z0+i*2*pi*f*m-i*Z0*cot(k0*d1);    % Impedance of air cushion (kg m-2 s-1).

X=(rho/sigma)*f;             % Delany and Bazley frequency parameter.
ZC=Z0*(1+0.0571*(X.^-0.754)-i*0.087*(X.^-0.732));
kC=k0.*(1+0.0978*(X.^-0.700)-i*0.189*(X.^-0.595));
m=rho*pi*b^2*(t+2*delta*b); % Mass of air block (kg).
Z2=ZC.*(Z1+i*ZC.*tan(kC*d2))./(ZC+i*Z1.*tan(kC*d2));
ZS=i*2*pi*f*m+Z2;
R=(ZS-Z0)./(ZS+Z0);          % Reflection coefficient.
a=1-abs(R).^2;               % Absorption coefficient.

semilogx(f,a)
title({['Helmholtz absorber']; ...
       [' d_1=' num2str(d1) 'm, d_2=' num2str(d2) 'm, t=' num2str(t) 'm, ' ...
       '\sigma=' num2str(sigma),' Pa s/m^2, b=' num2str(b) 'm, D=' num2str(D) 'm']})
xlabel('Frequency (Hz)')
ylabel('Absorption coefficient a')
grid on
axis([10 10000 0 1])
```

4. *References*

The following references are used.

B: Leo Beranek *Acoustics*, Reprinted 1996, Acoustical Society of America.

CA: Trevor Cox and Peter D’Antonio *Acoustic Absorbers and Diffusers*, 2nd Ed. 2009, Spon Press

K: Heinrich Kuttruff *Room Acoustics*, 5th Ed. 2009, Spon Press.

RW: Lennart Råde and Bertil Westergreen *Mathematics Handbook*, 5th Ed. 2004, Studentlitteratur.

R1: Tore Skogberg *Report 1 – Reverberation*, External project 2010.

Although the Internet cannot be used as a scientific reference, the *Uniform Resource Locator* (URL), i.e. the Internet “address” will be used for lightweight references.